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Open Quantum Relativity *

The relations between Quantum Mechanics and General Relativity are discussed in the light of a new approach, based on a covariant symplectic description of the two theories. Moreover, the necessity of a five dimensional space and of two time arrows comes out from the general assumption that the conservation laws can never be violated. The results of the comparison between the previsions of this theory and the experimental data of the most recent observations are presented.

1. Introduction

Open Quantum Relativity [3] is the attempt of a coherent dynamical theory, able to include in its scheme new phenomena and experimental observations, which are very difficult to include in the standard descriptions, because of the persisting difficulty in defining an unified vision of Quantum Mechanics and Relativity. This attempt is based on a principle: the General Conservation Principle, on a method: the dynamical deduction of the laws in an unitary frame with the minimal number of free variables and finally on a fundamental consideration: the possibility of a symplectic description for both Quantum Mechanics and General Relativity. In fact the contemporary physics offers a contradictory view, because, in this period of very rapid progress, and due of this rapidity, several new problems have been substantially removed and confined outside the mainstream of physics, even if they are no longer forgettable if we wish to carry on in this same progress. Let us remember some of them: a) the contradiction pointed out by the Einstein–Podolsky–Rosen (EPR) paradox [1], i.e. the possibility, under particular conditions, to per-

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turb a physical object without interacting with it in any known way; b) the existence of objects – the black holes – which, despite several attempts [2, 3], seem to violate the total energy conservation [4]; c) the consideration of quarks as elementary constituents even though it is generally hypothesized that probably they can never get their individuality [5]; d) the up to now observed absence of primary antimatter in our universe, despite the standard symmetric creation of matter and antimatter couples [6, 7, 8, 9]; e) the big bang theory, in both standard and inflationary cosmology, which is not yet satisfactory solved for the initial singularity [10]; f) the experimental results in quantum teleportation [11, 12] which suggest a $\Delta t = 0$ in transfer information time, in this way questioning the Relativity; g) the lack of an unitary description of all fundamental interactions [5]; and finally h) the fundamental question of the absence of an unitary theory [13, 14] connecting Quantum Mechanics and Relativity. Now, it seems to us that several of the contradictions arise from two principles, implicitly or explicitly contained in every modern scientific formulation. Principles which certainly can be considered reasonable, but that are common sense assumptions and not mathematical theorems: (i) the assumption of the existence of only one time arrow, from the past toward the future, which, starting from initial boundary conditions, leads in several cases either to a violation of the causality principle or at least to a new definition restrictive of its formal meaning [13, 14]; (ii) the assumption that only the geometry is given in the evolution of physical systems (as a typical example the invariance for diffeomorphisms) and any conceivable topology change is instead always considered as a «singularity». In other words the topology of the space–time manifolds, where systems evolve, is always given from the beginning and never considered as a dynamical structure.

From our point of view, these two implicit principles are leading to singularities, symmetry breakings and violations of conservation laws, and are responsible for the most significant contradictions of today physics. Moreover it seems to us that, in modern physics, the symmetry breaking and the violation of some conserved quantities are a sort of ad hoc hypotheses, invoked when a new phenomenon cannot be included in the standard schemes. In our opinion, a straightforward way to take into account the above issues and search for a new approach, in which the contradictions of modern physics can be solved, is deeply related to conservation laws, so than we propose the following point of view: to see what happens if conservation laws are *always* valid and symmetries are *always* maintained.

Directly due to the fact that conservation laws can never be violated, the symmetry of the theory leads to the very general consequence that backward and forward time evolution are both allowed and the necessary generalization of the approach to five dimensions, each one with real physical meaning, leads to the derivation of particle masses as a result of a process of embedding.

In doing this, we start with a discussion on the EPR paradox (initial point of the considerations involving Quantum Mechanics (QM) and Relativity); we proceed with the consideration of conservation laws and of the generation of two arrows of time, which can give rise to the entanglement of physical systems; than we generalize the approach to five dimensions in order to extend the time definition and to show how this leads to a scheme of unitary description of interactions; finally we present an unifying description of QM and Relativity and we match the results of the theory with observational data.

2. Quantum Mechanics and General Relativity. The role of Conservation laws

2.1. *EPR effect and Time Arrows*

The fundaments of QM, especially in its relations with the Relativity, are always broadly discussed, because the intrinsic characteristic of the QM is the existence of systems which have no definite value of some measurable quantities, if one does not measure them. The most significant point is that this is due to fact that the state of a system is a superposition of different states, and the only possibility left to the observation is to interact in an irreversible way with the system, changing its state with the measure process. This description, mainly due to Bohr, is known as Copenhagen Interpretation (CI), and, in it, the irreversibility of the measure process (and consequently the irreversibility of the time direction), plays a key role. Moreover the superposition of quantum states, that in CI is characteristic of the microscopic description of the nature, may have macroscopic effects and this is at the base of the EPR paradox. Let us suppose (taking the very clear Bohm example [15]) to have a spin zero particle, or in general a bound state, which decays into two particles each of spin 1/2. As far as the spin of the particles in a definite direction is concerned, the state of the system is described by a state vector of the following form:

$$| 0 \rangle = | \uparrow \rangle | \downarrow \rangle - | \downarrow \rangle | \uparrow \rangle \tag{1}$$

Where we must stress that every single particle is not a state by itself and the evolution of the state vector, with respect to the spatial distribution of the wave function, is not yet specified. Note that we do not perform measurement at time t such that $t_0 < t < t_m$ (t_0 is the time of decay and t_m is the time of the measurement) and therefore we do not know anything about the state of the system in that range of time. Now, at time t_m, we measure the spin σ_1 of the particle 1. The spin state of the system is still determined by (1), but now the measure operation causes the collapse of the wave function in one of the two states $|\uparrow\rangle\,|\downarrow\rangle$ or $|\downarrow\rangle\,|\uparrow\rangle$. This implies that, once the spin of particle 1 is measured, also the spin of particle 2 is instantaneously acquired, even if the particles are now far apart and not directly interacting. Therefore we have the following paradox: we have two particles in condition of absence of direct interaction, but the state of the system is such that, if we fix the spin state of one particle, also the spin state of the other particle is instantaneously fixed.

The non local behavior (connected with the Bell's inequalities) of this kind of systems has been tested by Aspect & others [16] and the experimental results show that the non locality is an actual feature of the nature. Let us summarize the results of this experiment (and many others) using Sakurai's words (in a book revised by J. Bell himself [14]): «all the experiments made, have conclusively shown that Bell's inequalities (which come from the "locality prescription" of Einstein) are violated and violated in a way which is compatible, within the errors, with Quantum Mechanic's prediction». To explain this paradox, so crucial since at the intersection between QM and Relativity, we have developed a new approach, starting from the general remark that the Noether Theorem [17] states that for every conservation law of Nature a symmetry *must* exist.

From this statement, *inside* the framework of conservation laws, it comes formally out the consideration of a backward time evolution of the wave function, since dynamics, if derived from a variational principle, is always symmetric under time reversal transformations. This general consideration indicates that conservation laws intrinsically contain forward and backward causation, even if against common sense and local realism, which instead assume just one arrow of time. Below we show, by very general arguments, that Bianchi's identities, which are geometric identities directly connected to conservation laws, contain symmetric dynamics. From such a dynamics, it is therefore possible to recover backward and forward evolution of the wave function, starting our considerations from the statement that quantum matter can be described by a scalar field ϕ [18, 19] and from a phenomenological definition of entanglement, as the phenomenon which takes place when two or more physical objects, despite being spatially disconnected, are subject to an inter-relation for which the effect of a perturbation on one of them induces a perturbation on the other one, without any direct interaction on each other. Let us now recall that, in a previous paper [21], it has been proposed a solution of the EPR paradox which do not formally change the frame of CI. This means that all the features of CI are preserved, but the microscopic time flow, for the wave function, is different from the time flow of the observer. As we outline, this possibility, although not intuitive and not referable to our macroscopic perception of reality, is compatible with Quantum Mechanics because of our intrinsic ignorance of the state of the system in the time interval $t_0 < t < t_m$.

The dynamics of a scalar field ϕ describing quantum matter, on a (curved) space-time, is given by the stress energy tensor

$$T_{\mu\nu} = \nabla_{\mu}\phi\nabla_{\nu}\phi - \frac{1}{2} g_{\mu\nu}\nabla_{\mu}\phi\nabla^{\mu}\phi + g_{\mu\nu}V(\phi) \tag{2}$$

which is a completely symmetric object.

Such a tensor has to satisfy the conservation laws

$$\nabla_{\mu} T^{\mu}_{\nu} = 0 \tag{3}$$

which are the contracted Bianchi identities for Tμν.

Sending to the quoted paper [21] for the full development, the final result is that the contracted Bianchi identities, if conserved, imply the Klein-Gordon equation which gives the dynamics of ϕ, that is

$$\nabla_{\mu} T^{\mu}_{\nu} = 0 \Leftrightarrow []\phi + \frac{dV}{d\phi} = 0 \tag{4}$$

where □ is the d'Alembert operator and $\nabla_{\mu}\phi^{\mu} = []\phi$.

It is interesting to note the full symmetry of the result, i.e. the Klein-Gordon operator is symmetric.

Specifying the problem to the case of a self-interacting massive particle, we can get

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \quad \text{and then} \quad \frac{dV}{d\phi} = m^2 \phi \tag{5}$$

so that we can write the Klein-Gordon equation as

$$([] + m^2)\phi = 0 \tag{6}$$

Being ϕ the scalar field, we want to stress that it can be interpreted as the product of two conjugate complex fields

$$\phi = \psi^{*}\psi \tag{7}$$

For consistency, the Klein-Gordon equation gives

$$([] + m^2)\phi = (\partial_{\alpha}\partial^{\alpha} + m^2)(\psi^{*}\psi) = (\partial - im)(\partial + im)(\psi^{*}\psi) = 0 \tag{8}$$

and Eq. (8) can be split, for massive particles, in the cases:

$$(\partial - im)\Psi = 0 \quad (\partial + im)\Psi^{*} = 0 \tag{9}$$

$$(\partial - im)\Psi^{*} = 0 \quad (\partial + im)\Psi = 0 \tag{10}$$

where $(\partial - im)\psi = 0$, can be considered as a «forward» propagator, while $(\partial + im)\psi^{*} = 0$ is a «backward» propagator and vice versa for the other two.

Instead, for massless particles, the four conditions reduce to the two ones

$$\partial\psi = 0 \quad , \quad \partial\psi^* = 0 \tag{11}$$

At the end, it comes out that a function (a superposition) of the form

$$\varphi(x) = \alpha_1\psi(x) + \alpha_2\psi^*(x) \tag{12}$$

where $\alpha_{1,2}$ are arbitrary constants, is a general solution of the dynamics and the states ψ and ψ^* can be considered, in our scheme, as *entangled* since they can influence each other also when they are disconnected. In other words, the absolute validity of conservation laws gives rise to a symmetric dynamics (backward and forward evolution of the system) and the entanglement of states is naturally determined without *any arbitrary violation*.

We want to stress that we find the four conditions (9-10) which satisfy Eq. (8), and this fact implies that backward and forward evolutions exist both for the Ψ-field and the conjugate ψ^*-field. In some sense, it seems that all the folds of light cone in Minkowski space-time have the same dignity, but we have been confined to investigate (at least macroscopically) just the fold toward the future (the arrow of time which we can normally perceive). These considerations are general and can be extended to curved space-time.

In order to save a logical connection in the fact that a measurement made on a particle seems able to affect the status of a particle disconnected far apart, it is important to notice that the statement can be re-expressed in terms of an effect of a measurement of a particle able to affect the system in the past: i.e. the (measured) relation of interference known as Bell inequalities, can be obtained if the particles decay in a state which depends on what will be measured.

2.2. *The General Conservation Principle in a five dimensional space*

Let us continue the description of this «different approach» and try to show how and why it seems able to take into account and solve several contradictions in Quantum Mechanics and Relativity [22, 23, 24]. Fundamental, at this point, is the introduction of the *General Conservation Principle*: «the fundamental conservation laws are the only ones having an absolute meaning and maintaining always their validity. This is the reason why, when it is otherwise impossible to maintain their validity, they determine the entanglement phenomenon, which allows in any case their recovery, thanks to a topology change and a related inversion of time arrows. So, the conservation laws are always preserved, the topologies not» [24]. This principle, due to the fact that mathematical formalism does not prevent backward and forward evolution in time, allows an explanation in physical terms of non-local behavior for entangled systems, open a discussion on the *first principles* leading to such symmetric time arrows and agrees with the experimentally observed violation of Bell inequalities and consequently of Einstein locality principle [13].

For the «traditional» causality principle [13] and Einstein–Podolsky–Rosen point of view [1], such a behavior is a paradox. Nevertheless, using backward and forward evolution on the same foot, entangled systems are naturally explained [21]. In other words, if we maintain conservation laws absolutely valid, we *need* two arrows of time, and backward evolution turns out to be a feature of nature, which we cannot ordinarily feel, (being confined in the forward fold of light-cone) but which emerges as soon as a conservation law has no other way to maintain its validity. As a consequence of this view, strictly related to the concept of entanglement, there are topology changes taking place in order to preserve a conservation law, e.g., matter–energy in black hole dynamics, or quantum numbers in EPR effects [21]. In the first case, as we will see in the following, the entangled system is constituted by a black hole which dynamically evolves in a white hole through a topology change (the real essence of a wormhole). The general feature of such a result is that the conservation laws can never be violated, while topologies can change and so they can be considered a dynamical quantity. Moreover the hypothesized existence of backward and forward evolution leads to the necessity of a fifth dimension. Let us remember a very simple example, imagining a hypothetical one-dimensional universe. In such a universe, a point placed to the left of another one could never invert its position exchanging the left with the right, while if we pass to a 2-dimensional universe it becomes obviously possible to exchange the relative position of the two. This is true also passing from four to five dimensions (making possible the evolution in both directions of the time axis) [22, 23, 24]. But this is not, in our mind, just a technicality, because we think in terms of a 5-dimensional universe with real physical meaning, as we will show. In our approach, as the fourth dimension can be naturally related to time in General and Special Relativity, the fifth dimension can be naturally related to the masses of particles (so possibly giving rise to the differentiation of fundamental interactions [24, 30]). In other words, we can deal with a generalized five-dimensional *masschronotope*, every point of which is labeled by space, time and mass. It is relevant to note the fact that we do not perceive the fourth time-dimension on the same foot of the space-dimensions and the situation is analogue for this fifth mass-dimension. The set of conditions (9) and (10) are fundamental in our approach, since they give rise to forward and backward dynamics. From a relativistic point of view, in the cases (9) and (10) we are *inside* one of the folds of the light-cone, in the case (11) we are, instead, *on* the null surface and the role in distinguishing dynamics is played by the mass of particles [30]. The distinction of backward and forward dynamics depends on the dimension of the space–time we are dealing with, in a 5-dimensional manifold, all particles are moving without preference of backward or forward evolution, while it is the embedding procedure from 5- to 4-dimensional space–time which gives rise to two dynamics (i.e. two arrows of time), but conservation laws are preserved in any case, also in the process of mass generation. In conclusion, in the light of the above considerations, the concepts of entanglement and topology change are features of the theory which emerge in order to preserve conservation laws [24]. In previous papers, we have treated the problem for different and general cases. In [21], we showed that EPR paradox can be solved under the standard of this *General Conservation Principle* since the paradox is solved by an entangled superposition of forward and backward solutions. The we have taken into account an astrophysical system like a black hole [33]. Assuming, as natural law, that the collapsing matter–energy is totally conserved, such a black hole evolves, trough a topology change (the structural mechanism of the so-called wormhole) in a white hole. The black hole and the white hole (in fact a white fountain) are so two entangled objects which can live in two causally disconnected space-time regions. All these indications seem to suggest that the conservation laws show the same general validity, so they seem all related by a *unique, general and fundamental conservation principle*. Finally, in this approach, the possibility to evolve backward to the past (i.e., in what we perceive as a past) is a general feature of nature, but in our «forward» fold of light-cone, we cannot directly experience it. The only way to perceive the backward evolution is under the extreme conditions in which a conservation law, being otherwise violated, determines a space–time breaking via a topology change connecting two entangled systems and this phenomenon is allowed by the presence of a fifth dimension of real physical meaning. It therefore appears that several shortcomings of current physics can be solved assuming this *General Conservation Principle* always valid and this approach seems suggested by nature itself. We want to outline that this is a general result, because the connection among conservation laws, symmetries and first integrals of motion have a deep physical meaning.

2.3. *Causality, Entanglement and Topology change*

As it is well known, Einstein, Podolsky and Rosen stated that Quantum Mechanics, contradicting the locality principle, leads to results which violates the causality principle [1]. In the new approach, the complete redefinition of the entanglement concept, based on the impossibility of violation of conservation laws and on the generalisation to a five dimensional space, leads instead to the recovery of the classical and fundamental idea for physics of Causality. The overall validity of the conservations laws induces, in the cases where they would be otherwise violated, topology changes which make possible «tunnels» (like wormholes in astrophysics) connecting separated space-times regions. This phenomenon will be shown to be a-luminal and able to open the door, without overcoming the light speed, to a conceivable time-machine. In fact, QM asks for that an interaction, even if only on a single part of a quantum system, determines a dependent evolution of the correlated quantities of the other part, also when these parts are placed in regions «causally disconnected» of space-time, (i.e. when they are so far that no direct interaction between the parts can occur, in the light speed limit). From the very beginning Schroedinger tryed to overcome the problem with the «qualitative concept» of entaglement, described as a sort of deep connection, not yet dynamically specified, able to relies two causally disconnected, but quantum related, object. Starting from the qualitative concept of Schroedinger, later accepted and elaborated by many authors [28], let us rephrase here the statement (developed in work [20]), on the causality in this new entanglement definition:

«Two states, *spatially separated and causally disconnected in the four dimensional space that we can ordinarily perceive*, are entangled if an interaction with one of them can influence the other one, without in any way directly interact with it, *because a four dimensional entanglement means that a causality nexus exists in a larger five dimensional physical space*».

The reasoning leading to this definition seems a real necessity, because the entanglement, concept unavoidable to explain why Quantum Mechanics works, would be otherwise simply impossible, without violating the causality principle and also the logic, which states that «it is impossible an interaction with an object without in some way interacting with it».

If we do not hypothesize another – and really physical – dimension, in which the two states are causally connected, – *so then restoring the causality principle* –, what we call entanglement would remain a «necessary but impossible» phenomenon.

In our theory, the conservation laws are the first principle which determines all the following evolution, since the fact that they can never be violated leads to a mechanism to avoid such a violation, also in the cases in which, for the standard interpretations, the violation should occur. This mechanism is the *topology change*. It is the topology change which provides the dynamics allowing the very particular interaction between two otherwise causally disconnected states, that we call entanglement, it is the topology change which makes possible the «hole» connecting the top and the bottom of the Fig. 1, or, more specifically, the formation of a wormhole connecting a black hole with its entangled white hole [33], through a Goedel type change of topology [29]. Entanglement, in this picture, maintains the meaning of the underlying mechanism allowing interactions otherwise impossible, which Schroedinger initially gave to it, but with a dynamics providing an explanation of the phenomenon.

This entanglement, even if allowed by the existence of a 5D-space, nevertheless gets its full meaning in four dimensions, because it is a mechanism able to provide a description of a peculiar phenomenon otherwise very difficult to explain, but loosing its necessity in five dimensional space, because, in this totally connected space, *all* the interactions recover naturally the causality. To recover the general validity of the conservations laws leading to a General Conservation Principle, the necessity of topology changes and of two time arrows (instead of the ordinary one, that we can naturally perceive), as well as the existence of a fifth dimension, are strictly necessary.

Sending to the quoted papers for the general description and the formal derivation of these results, here we want to point out that the above considerations bring to a very deep meaning of the entanglement concept, solving the EPR paradox without disproving Quantum Mechanics and, at the same time, recovering the Causality Principle.

2.4. *Contemporaneity, entanglement and light speed*

In the light of the new approach, another very important question, coming always from the EPR effect find a possible solution. The question of the hypothesized contemporaneousness of the effects of an interaction on one part of a system, with the induced effect on the other part (placed far and non directly interacting, but entangled), question becomes recently even more important after results seriously claiming for instantaneous quantum teleportation [11, 12]. The important point is the possibilities to really transfer information, which are instantaneous and impossible to be detected outside the entangled emitter-receiver system. The long series of Bob & Alice (receiver & emitter) papers is the most famous example of it [11] and a special relevance deserves the very important claim by Bouwmeester et al. [12], where, following Bennett et al. suggestion [35]: «It is possible to transfer the quantum state of a particle into another particle, provided one does not get any information about the state in the course of the transformation», they presented experimental evidences of an effect of polarization, given on one of a couple of entangled photons, transferred on another one without any direct standard interaction between them. The experience is ultimately an «entanglement swapping», from a photon of an entangled couple to a third one and, at the end, photon 1 is no longer available in the original state, but photon 3 is now in that state and this is not a clone but really the result of teleportation. It remains the fact that it is necessary to send to Bob, in standard way, the information of the state of the entanglement (one possibility among four) between photon 1 and photon 2, and this fact opens a big discussion on the real meaning of teleportation. Anyway, besides the possible and very important applications in computing sciences, these experimental results deserve some important comments: i) it seems possible to determine the entanglement also between particles (and very probably also between more complicated objects) without the same origin, so opening the doors to a conceivable general technique; ii) it seems possible to send information, via entanglement, without any destroying influence of the environment and as far as one wants (even without knowing where); iii) it seems possible to send information instantaneously, even if a conventional message is necessary to inform and check.

The last two points seem to indicate a $\Delta t = 0$ in the transfer operation, the only one way to send information instantaneously (as far as we want) and without any environmental influence. This really seems in contradiction with Special Relativity, even if some authors try to avoid such a contradiction saying that, due to the necessity to send also a conventional message, this would restrict again the phenomenon in the limits of Relativity, because it could not be used to send information faster than light. Nevertheless but this last reasoning is not fully convincing, because, despite our capability to utilize the information only in conventional way, it remains the fact that the phenomenon would be instantaneous.

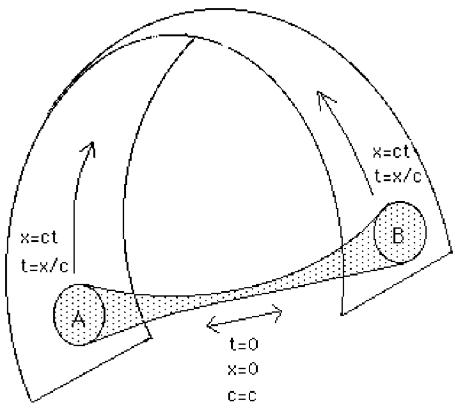
This question, despite the attempt to restrict its meaning, remains a major problem, because of, due to the hypothesized contemporaneity, it seems to mean the possibility to travel faster than light speed limit, in clear contradiction with the basis of the Relativity, evidently proved as correct, but this is a problem only if treated in terms of super-luminality, whereas it is no longer a puzzle if treated in terms of *a-luminality*.

Looking at the Fig. 1, the question finds a possible solution.

Let us take into account (Fig. 1) a bounded surface: it is evident that it would take time to go from a point at the upper side to the corresponding image point at the lower side of such a surface, but this time is reduced to zero, if a mechanism exists to make an «hole» and get directly, once defined an orientation, the bottom from the top. It is straightforward to see, once defined the transformation for time and space intervals $\Delta t'$ and $\Delta x'$:

$$\Delta t' = \Delta t(1 - v^2/c^2)^{1/2}, \quad \Delta x' = \Delta x(1 - v^2/c^2)^{1/2} \tag{13}$$

that the reduction to zero of the space interval $\Delta x'$ implies, in the second of the (13), that v = c , so then $\Delta t' = 0$. The situation is that one depicted in Fig. 1, the travel is from A to B (or vice versa), the mechanism is the change of topology and



= 4 Dimensional Manifold

U = 5 Dimensional Manifold

Fig. 1.

the deep reason of that is the physical necessity to save the conservation laws, without overcoming the light speed. This type of travel is an *a-luminal* travel.

In our theory, the conservation laws are the first principle which determines all the following evolution, through the topology change which makes possible, for example, the «hole» connecting the top and the bottom of the above figure, but the fundamental concept that has to be stressed here, is the «a-luminality», because it saves contemporarily conservation laws and causality principle, and it seems moreover the only one able to be compatible with the necessity to extend, but not contradict, the Relativity.

2.5. *A covariant symplectic structure for Quantum Mechanics and Relativity*

In order to formulate the General Conservation Principle at fundamental level, it is necessary to search for a covariant symplectic structure, starting from invariant quantities. It is well known that all the attempt of quantization of General Relativity, till now, get only non resolutive or contradictory results, so that a quantum theory of gravitation does not still exist in closed and self-consistent form.

For example in the Arnowitt, Deser and Misner (ADM) formulation of the gravitational field, the authors get a Hamiltonian definition of the field, but with a reduction (3+1) of the dynamics, where time is a priori distinct from the spatial degrees of freedom. In that approach the field equations con be considered as Hamilton equations, but nevertheless, in such a way it vanishes the most fundamental feature of General Relativity: the general covariance. In this sense a canonical formulation of gravitational field is not covariant. Another contradiction is that it is not possible to define a quantum field theory in which the dynamical degrees of freedom, the metric tensor $g\mu\nu$ and the background coincide, as it is the case of gravitational field. Starting from these considerations, it is necessary to search for invariant structures, always conserved, capable of giving rise to a symplectic structure, independently of metric tensor and allowing to formulate covariant Hamilton equations. In this way the General Conservation Principle can be formulated at fundamental level, so leading toward a general quantization scheme. In this perspective, it is possible to build Hamiltonian invariants starting from covariant and contravariant vectors, bivectors and, in general, tensors appropriately contracted. The spurious variation of these quantities are intrinsically conserved and, taking into account the covariant derivatives of the component vectors, it is possible to obtain a covariant symplectic structure which naturally lead to Poisson brackets and thus Hamilton equations. The key ingredients of this approach are the Hamiltonian invariants appropriately built and the affine connections (not the metric), which are giving rise, through the covariant derivation, to the Hamilton equations. And this covariant symplectic scheme is completely general. Through the specification of the vectors (or tensors) which constitute the Hamiltonian invariant, it is possible to get every specific field theory, because the scheme is of general validity and can be applied to any field. For instance, if the vectors are the four-velocities on curved space-times, the two Hamilton equations correspond to the equation of geodesics and to that of geodesic deviation (the equations for q_i and p_i of the usual Hamiltonian mechanics), while if the vectors are four-potentials of the of the electromagnetic field, we obtain the Lorentz gauge and the Maxwell equations.

3. The role of Time in Modern Physics

3.1. *Time in the framework of an Extended Relativity*

Even if it can be considered the final synthesis which solved many problems of classical physics, General Relativity is, above all, the cornerstone of a new way to see the nature and opened the door to a different way to see and define also old concepts, especially the time. In fact, if the possibility of «relative» time travels in the future is certainly possible already in the General Relativity framework (*e.g.*, for an astronaut in the future of the human kind remaining on Earth, if accelerated for a significant period in a rocket outside the Earth itself) it is in the post relativistic theories that the time travel hypothesis takes a more general meaning, mainly if such theories are taking into account also effects coming from Quantum Mechanics. If we fix now our attention on the possibility (induced by conservation laws induced) of change of topology, we can get, as an example, a Goedel type condition [29], of a closed time-like curve (CTC) geometry, recovering its deepest meaning of opening the perspective to travel even back in time. In fact a closed time line, which Goedel first presented as purely formal possibility in an unconventional solution of Einstein equations, (simplifying, a cylindrical coordinates choice where the longitudinal-one is spatial and the circular is temporal, instead of the usual contrary) makes it possible to track back in time the footing done, so than «to pass again» in the past. Now, what for Goedel was only a mathematical picture, here becomes, for the first time a real physical hypothesis, because based on a dynamics (the forward and backward evolution of the split Klein-Gordon equation), on a «necessity» (to save the conservation laws and the causality principle), on a conceivable footing (the induced topology change defined in an appropriate mathematical domain) and finally on a theory able to propose an explanation for several physics open problems (EPR paradox, entangled teleportation, black hole existence, gamma ray bursts) without contradicting the light speed limit. After Goedel, many authors showed that, far to be an exception, the possible existence of natural CTCs in the universe should be an infinity, unless the existence of hypothetical rules (like, for instance, the Chronology Protection) forbidding it. Nevertheless, up to now, the proposed rules are based on semi-classical considerations, so then no longer holding in the framework of a full quantum gravity formulation, because they are an inappropriate combination of quantum matter field in a classical space-time leving valid the CTC hypothesis [38]. Let us now enter into some general properties of space-times with CTCs [38, 39, 40, 41], first remembering that solutions of Einstein's equations allowing CTCs have been known from a long time. The earliest example of such a space-time is a solution obtained by Van Stockum in 1937 [47], which describes an infinitely long cylinder of rigidly and rapidly rotating dust. Another important example and perhaps the most famous, is Goedel solution [29] representing a stationary homogeneous universe with non-zero cosmological constant, filled with rotating dust. CTCs are also present in the interior of Kerr black hole in the vicinity of its ring singularity, and other examples of spacetimes with CTCs are discussed by de Felice [43]. In the general case, a space-time can be divided into *chronal regions*, without CTCs, and *achronal regions* which contain CTCs. The boundaries between the chronal and achronal regions are formed by *chronology horizons*, precisely a chronal region ends and an achronal region begins at a *future chronology horizon*, while an achronal region ends and a chronal region begins at *past chronology horizon*. Thus, achronal regions are intersections of the regions bounded by both of these horizons. In the framework of General Relativity, in order to create a *time machine* (which, in this context, means in general a region with CTCs) by using a wormhole, one needs to assume that there exists the possibility, in principle, to make it long living and traversable, which, in standard view, needs the violation of the averaged null energy condition [48]. After all the whole time conception, only apparently intuitive, is perhaps so dramatically changing in the framework of post relativistic theories, that all our knowledge of physics will be deeply modified.

3.2. *Toward a new time definition*

Let us now go deeper in the new approach, which is starting from the main stream, and so in the framework of the standard General Relativity and Quantum Mechanics, but providing a new point of view in the light of the *General Conservation Principle* [24], principle which leads to a new class of post-relativistic theories. The first characteristic of this approach is that it is not violating Special and General Relativity but it is extending their range to include backward time solutions as a «necessity» to preserve always and in any case the conservation laws and the causality principle. The Many Worlds interpretation of Quantum Mechanics [42] is also a necessary ingredient of this approach, because it comes out as a consequence of a process which starts from basic principles. Namely: the solution of the EPR paradox as a conflict between Relativity and Quantum Mechanics [21]; the generalization of entanglement concept as a gate, through topology changes, for motion in time [20]; a dynamical scheme for the unification of the different interactions [30]; and finally a description, through a covariant symplectic structure, of both Quantum Mechanics and General Relativity [46, 50]. In this sense, we speak in terms of Open Quantum Relativity and General Conservation Principle.

We should stress that the whole question of time, and of time machine hypothesis, is completely changed by this new theory, because «moving in time» is no longer an extreme possibility only due to a relative motion of two frameworks on the same time arrow, but a more general possibility due to the existence of a second and backward directed time arrow. In the «conventional» time machine, «backward in time» means only a relative or circular past, while, in our theory, there is also another very general possibility and the real puzzling question is how to get «physically» a backward time arrow, that we cannot ordinarily perceive, but which has to exist [20, 21, 22, 46].

In order to substantiate the previous statement, let us recall the equations describing quantum matter:

$$\psi(x) = e^{-ikx}\, u(k) \quad , \quad \psi^{*}\,(x) = e^{ikx}\, u^{*}(k) \tag{14}$$

which, as we have seen, can be interpreted respectively, as progressive and regressive solutions in four combination. It comes out that a function (a superposition) of the form

$$\varphi(x) = \alpha_1\psi(x) + \alpha_2\psi^{*}(x) \tag{15}$$

where $\alpha_{1,2}$ are arbitrary constants, is a general solution of the dynamics and the states ψ and ψ^{*} can be interpreted as *entangled* since they can influence each other also when they are, as standard, considered disconnected. As we will see, the resulting scheme shows itself properly working in explaining many paradoxes and shortcomings of modern physics, with the minimal necessary number of parameters in comparison to all existing theories (like Strings, or Supergravity).

Particularly interesting, in the context we are discussing here, is the fact that *entangled* gravitational systems constituted by black holes, wormholes and white holes naturally emerge, through topology changes, starting from the request that the mass-energy of collapsing systems is conserved in the framework of the General Conservation Principle. The main point of this result is that such systems can be stable so that (thanks to conservation principle which avoids spontaneous symmetry breakings) time travels and time machine become (at least on a theoretical ground) a conceivable possibility [33] (moreover, starting from 5D dynamics, CTCs are ordinary solutions of field equations).

These results, in the context of time travels and time machine, have a deep meaning since are not «anomalies» in the framework of a standard theory (like General Relativity) but are «ordinary» outputs in the framework of an «Open Quantum Relativity» (OQR).

In fact, while the concept of time travels in General Relativity is only related to the difference in space-time among different frameworks, here, in the new approach, the relative difference can be directly taken into account on two time arrows and this means that a closed time-like path does not need, to exist, a topology change imposed «by hand» (like in the Goedel picture), because the projected motion along a circumference is the combination of two linear motions in opposite sense, so then the topology change is the natural result of an underlying feature of Nature. This feature comes in evidence every time a change of topology happens, and this occurs if a conservation law would be otherwise violated, providing dynamics of what Schroedinger called entanglement.

It is evident that there is the full recovering of General and Special Relativity (because we do not need to travel faster than light) but the mechanism of relative motion between two frameworks, which is generating the hypothesis of time travels, is clearly not the same if applied to one or two time arrows. In this sense, we deal with «Open» Quantum Relativity. The new theory, even if not giving any suggestion for the technical devices of a hypothetical time machine, provides nevertheless first principles on which such a machine should be based, because it is fixing the limits in which such phenomenon should be perhaps possible. In fact, only in a situation of impossibility to avoid a violation of a conservation law, it happens that the nature reacts changing the topology, so then we should find in the universe such a situation or be able to recreate it. In the first case (for the today knowledge), the «laboratory» for such an experiment should be a black hole, in the second case, an «entanglement machine». This would be a kind of machine not possible to design yet, but that we cannot in principle exclude. In fact, there are works of authors [44] which demonstrate that an entanglement of two macroscopic systems does not need the entanglement of every component of them in correspondence one-to-one (which will be probably impossible to obtain) but only a correspondence at least under the limit of the uncertainty principle (certainly not easy, but perhaps not impossible). In this framework, the Many Worlds Theory [45] of an infinity of universes, should be taken not as a possibility, but as a necessity, in order to avoid new and deeper paradoxes, induced by backward time travels (like the famous hypothesis of a time traveler killing his grandmother when she was young before his own birth, so destroying himself and making impossible to kill grandmother). Only the Many Worlds Theory, can settle this puzzle, saving general time travel hypothesis, causality and logic together. It is very difficult, today, to estimate all the consequences of the above considerations concerning a «time machine», nevertheless one should derive some conclusions to open an useful debate.

Time traveling hypothesis, coming out from a theory in which two time arrows and CTCs are general features of the Nature, represents a break with the more traditional points of view, but able in principle to reconcile General Relativity and Causality in a framework in which Relativity and Quantum Mechanics are deeply connected.

3.3. *The new approach and the experimental observations*

This approach, based on first principles, is of general meaning, because, thanks to the consideration of the conservation laws as absolutely valid, suggests the existence of a real five-dimensional space and of two time arrows as necessary charatheristic of the Nature, independently of our capability of perceiving them. The resulting scheme is internally coherent and seems able to solve several problems of contemporary physics, far beyond the starting point of the EPR paradox, because several experimental evidences comes out from cosmology and astrophysics.

For instance, in our scheme, the search of the initial conditions of the Universe evolution leads to the significant result that the existence of conservation laws dynamically determines that the so called «Wave Function of the Universe» selects first integrals of motion whose cosmological solutions are «observable universes» (in the same sense of space-times classically considered [37]), so proposing a solution for an old problem [42, 45]. This theoretical approach is confirmed by cosmological observations, because the model coming out from the reduction process from 5 to 4 dimensions largely matches the recent observational results. In fact, combining data from Supernovae of Ia type, from cosmic background radiation (in particular COBE, BOOMERANG and WMAP), from Lyα-galactic clouds and from the large scale galaxies clusters distribution [32], it comes out a flat cosmological model, whose dynamics is dominated by 30% of matter (baryonic and dark) and by 70% of dark energy (which can be considered either cosmological constant, or the so called Quintessence scalar field, or some other unidentified dynamical entity). The main characteristic of this new observational frame is that the Universe seems to accelerate, instead of the contrary as in Standard Cosmological Model predictions, and moreover the possible sources of the dynamics remain totally unknown. With our mechanism, instead, we succeed in reproducing the cosmological parameters in agreement with the observed values, and this starting from a general theory without arbitrarily introduced conditions. In particular, taking into account our cosmological model [31] derived from the embedding process from 5 to 4 dimensions, we obtain $\Omega_M \approx 0.3$ (density parameter of baryonic and dark matter), $\Omega_\Lambda \approx 0.7$ (density parameter of dark energy), $q_0 \approx -0.5$ (deceleration parameter), $t_0 \approx 14$ Gyrs (estimated age of the Universe), $H_0 \approx 65$ Km / sec / Mpc (Hubble parameter) so reproducing correctly the observed values of the five more significant cosmological parameters. We should stress that also the scalar field, whose dynamics is ruled by the Klein-Gordon equation (i.e., as we have seen, by the Bianchi identities) is recovered in our frame. Another astrophysical problem which can be settled in this scheme, is that one of the dark matter that should occur to explain the observed dynamics of the rotation curves of the galaxies. In fact, the effective theory of gravitation, emerging from the process of dimensional embedding, implies a variable gravitational coupling, in which the Newton constant results to be a parameter dependent from the scale. In other words, the Newtonian potential, able to well describe the gravitational effects up to Solar System scale, have to be corrected by Yukawa type terms, depending on characteristic lengths, like for instance the typical scale of galaxies (from 10 to 100 Kpc).

In this scheme the enormous quantities of dark matter necessary to explain the observed dynamics of galaxies (up to 90% of the total mass of a typical galaxy) are no longer necessary, because the scale dependence of the gravitational interaction can provide a correct potential, without the addition of elements still unidentified. Two other astrophysical problems can find a possible solution in the scheme of Open Quantum Relativity: black holes and gamma rays bursts [33]. In fact those extremely energetic bursts of Gamma Rays, up to now without a well established mechanism able to explain their origin, power, polarisation and spatial distribution, in the light of the new theory can be considered as the signature of dynamical process of entangled astrophysical systems, black hole – wormhole – white hole. In the frame of OQR, the idea is that the absolute conservation of the mass-energy, during the gravitational collapse (black hole), induces a topology change (wormhole) in such a way that, in another space-time zone, it emerges an energy fountain (white hole).

This process can provoke an explosive emission (peak of the gamma ray burst) with a following tail (spectrum of gamma ray burst). From the model, it emerges that if the black hole is generated by a star big enough (for example of the order of 10 solar masses) it is possible to obtain the energetic values of gamma ray bursts. Moreover, the gamma ray bursts distribution seems like to the distribution of the evolved stars (i.e. the stars able to generate black holes) and finally the observed polarization can be naturally explained thanks to the passage of the energetic flux through the wormhole [33]. Finally a recent and interesting result is the correct reproduction of the observational limits of neutrino oscillations, derived from the induced gravitational field in our theoretical scheme [51]. As a conclusion of this short review of experimental evidences, it seems that several open problems of the contemporary physics, can be coherently explained (without elements arbitrarily introduced) thanks to the General Conservation Principle in the frame of the Open Quantum Relativity.

4. Discussion and Conclusions

Open Quantum Relativity [46] is the attempt of a coherent dynamical theory, capable of including in its scheme new phenomena very difficult to be framed in the standard descriptions, and based on a General Conservation Principle and on a covariant symplectic description for both Quantum Mechanics and General Relativity.

The scheme is the following:

i) We consider that the conservation laws are always valid, i.e. they can never be violated [24], so then symmetries are conserved.

ii) The framework in which the new approach takes place, is a five dimensional physical space [23, 24].

iii) A correlation between two objects causally disconnected (i.e. so far from each other that they cannot interact in the limit of the light speed) means that they are entangled, i.e. causally connected in a space scaled up to five dimensions [23, 24]. This means, in the ordinary four-dimensional space-time, that the causality can be generally recovered, admitting that the present can be influenced also by the future and furthermore that the contemporaneity of interaction effects on entangled objects is an a-luminal process.

iv) In five dimensions, backward and forward time evolution are both allowed and without distinction (the fifth dimension makes the time free). While, in our 4D space-time, it emerges the distinction in two time arrows, together with our limitation to perceive only one of them, except for the effects of an entaglement condition trough a topology change [23, 24].

v) The possibility to describe the Relativity at a fundamental level, is based on the identification of a symplectic structure common to both Relativity and Quantum Mechanics, so allowing to formulate the Poisson brackets of affine connections, instead of a canonical quantization of the metric degrees of freedom [50].

If the direct result of this theory, as we have seen, is to propose an explanation of several unresolved physical problems in a coherent scheme, there are also important long term consequences which derive from such an approach on time definition. These consequences have a deep meaning on the hypothesis of «Time Machine», because they are no longer anomalous possibilities in the frame of a standard theory (the General Relativity) but «ordinary consequences» in the frame of the Open Quantum Relativity. In fact, in the new approach a CTC, in order to exist, does not need extremes boundary conditions, because it is possible to obtain the motion along a time circumference as a combination of two opposite linear motions. This behaviour comes in evidence every time a change of topology happens, giving the dynamics of what Schroedinger called entanglement. The new theory, even not giving any suggestions on the techniques of a hypothetical time machine, it gives nevertheless the principles on what such a machine should be based on, because it fixes the limits where such a phenomenon could be conceivable [22]. In fact, only in a situation of impossibility to avoid the violation of a conservation law, it happens that the nature reacts changing topology, so then it occurs first of all to find in the universe such a situation or in alternative, to be able to recreate it in laboratory. In the first case, the natural place for such an experiment would be a black hole, in the second an «entanglement machine», which is a type of machine still of unknown feature, but perhaps not impossible to conceive [49]. In this picture the Many Worlds theory have to be considered because only this theory in its full version [45], hypothesizing an infinity of universes (each one of them representing a virtual possibility which can become real) can settle this frame, saving the general hypotesis of time travels, causality and logic together. So, in this approach, each universe can be hypothesized as a «local fact».

It is very difficult, today, to estimate all the consequences of the previous considerations, and mainly those concerning a time machine, nevertheless one can try to derive same conclusions to open an usefull debate. The time travel hypothesis, coming out from a new theory in which two time arrows and CTC's are considered a general characteristic of the Nature, it represent a breaking point with the traditional point of view, capable, in principle, of reconciling General Relativity and causality in a frame in which Relativity and Quantum Mechanics are deeply connected. The question of time as relative entity, is not ceasing of changing physics, starting from the initial definition in Classical Mechanics and then in Relativity, up to today conceptions and it seems that their effects are capable of producing even more effects in future. In fact, the existence of experimental evidences not only for well known standard behaviours (like the lengthening of the life of accelerated particles), but also for absolutely new effects (like the quantum teleportation through the entanglement) makes impossible to simply close the door to any hypothesis of time tunnelling and so, as a consequence, it comes out the necessity of a new theoretical scheme including also this phenomenon. We are in the middle of a critical change, we must complete the passing and find a synthesis of the discoveries of last century, which are tesserae of a mosaic still incomplete, but that in any case can not be settled just with phenomenological or semi-classical approximations. As many others, we have considered the necessity to search for a Quantum Relativity as an attempt impossible to delay, because related to all possible solutions of modern paradoxes and shortcomings, which emerges from first principles of physics.

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