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Maxwell's Dimensional Approach to the Velocity of Light: Rise and Fall of a Paradigma

1. Introduction

Maxwell's contributions to physics have been extensively scrutinized by historians in the last decade. Certain aspects of his work, however, are still partially unexplored. An example is, in my opinion, Maxwell's electromagnetic theory of light and, in particular, the metrological and dimensional theories which represent a large part of its supporting evidence. In fact these theories are used by Maxwell in order to prove the equality between Weber's conversion factor and the velocity of light, one of the main evidence in favour of his optical theory.

It is known that Maxwell brought into his electromagnetic theory of light concepts and data which were obtained in the 1850's by the German Physicist Wilhelm Eduard Weber (1804-1891). Maxwell's conclusion was that Weber's conversion factor, which in Weber's theory corresponded to a relative velocity of motion of electric particles, was representing, in Maxwell's theory, the velocity of electromagnetic waves and of light. A momentous conclusion because it transformed a velocity of motion and a relative velocity in a propagation and absolute velocity!

The conceptual difficulties presented by this transformation are documented in Maxwell's papers and specifically in the many routes he followed in the course of his scientific carear in order to reach a satisfactory demonstration for it.

The proof of the electromagnetic theory of light is assigned by Maxwell to arguments of various kind, but the most important one is the demonstration that electromagnetic waves propagate in the aetherial medium with velocity of light. This demonstration is presented in different forms along with the development of Maxwell's electromagnetic theory, since the first paper he devoted to this problem in 1862, until his 1873 masterwork: *A Treatise on Electricity and Magnetism*.

In all of these works Maxwells is faced with the problem of checking a theoretical value of the velocity of the electromagnetic waves, calculated by his

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electromagnetic field theory, against the velocity of light, known to him because of measurements made by Fizeau (1849), Foucault (1862), and also by the ones related to the aberration of light. An independent measurement of the velocity of the electromagnetic waves was, in Maxwell's time, beyond the reach of his experimental technique. Heinrich Hertz, as it is known, experimented and measured the velocity in question, eight years after Maxwell's death. Maxwell however succeeded just the same in measuring indirectly this velocity and found that it was approximatedly equal to the velocity of light.

In this paper I am primarily concerned with Maxwell's indirect procedure in measuring, throught Weber's factor, the velocity of his electromagnetic waves and with the difficulties he faced in reaching a satisfactory demonstration. These difficulties are mainly due to the fact that, in Weber's theory, the existence of Weber's factor, *i.e.* of a ratio between the two measures in electrostatic and in electromagnetic units of the same charge, was a direct consequence of those parts of Weber's theory, the foundamental law, and expecially the convective conception of currents, which Maxwell did not accept. Due to this, Maxwell introduced in his theory Weber's factor as a consequence of dimensional equations.

In may view then, Maxwell's recourse to dimensional theories, in his various demonstration of the equality between Weber's factor and the velocity of light, was mainly justified by his refusal to accept into his electromagnetic theory the " convective hypothesis of current ", which he labelled in his *Treatise* as an " exceedingly artificial hypothesis ". If one considers that the convective conception of electric current has been, since Lorents's time at least, the main pillar of any modern theory in Electrodynamics, one can realize how, through this absence, Maxwell's theory was deeply typified.

Maxwell's historians have perhaps favoured those parts of Maxwell's work which are, more or less, related to our modern theory. The consideration of some outmoded or controversial parts of his theories, such as the ones dealt with in this paper, will contribute, I hope, to a better understanding of the historical collocation of Maxwell's Electromagnetism.

2. Maxwell's various approaches for a solution to the problem of the equality between Weber's factor and the velocity of light

The antefact is that Weber in his theory (1) defined a system of Absolute Units for the electric and magnetic quantities and introduced the concept of a characteristic velocity of motion of electric particles, which in 1856 he measured in form of a " conversion factor " between electric units, thanks to the aid of

(1) W. Weber, " Elektrodynamische Maasbestimmungen. Über ein allgemeines Grundgesetz der Elektrischen Wirkung " (1846), in W. Weber, *Werke*, Berlin 1893, Dritten Band, 25-211.

his colleague, the experimentalist Rudolf Kohlrausch. Together they found (2) this factor to be approximately equal numerically to the velocity of light, but to this result the two German physicists did not attach any significance as a possible hint for the unification of light and electricity. On the other hand, Maxwell developed a theory in which it was shown that Weber's factor was in fact the velocity of light.

It may be helpfull, for the understanding of what follows, to notice that all the various solutions given by Maxwell in his scientific papers to the problem of Weber's ratio or conversion factor, can be considered as formed by two distinct parts: the first one (I call it Part A), connects the velocity of the electromagnetic waves to the aethereal constant k and μ, the "dielectric capacity" and the "magnetic permeability" of the electromagnetic aether. The other one, (Part B), connects the same aetherial constants to Weber's ratio or conversion factor. The correlation between Part A and B, is required to prove that the electromagnetic-waves velocity is equal to Weber's ratio. Since Weber's ratio is numerically equal to the velocity of light, Maxwell thus proves that electromagnetic waves propagate at the speed of light. This correlation constitutes in fact the indirect measurement for waves' velocity which above I hinted about.

The development of the complex set of conceptions and theories which largely deals with Part A, has been adequately analized in recent years by historians (3); however Part B has untill now escaped their attention, and I think that this Part also offers interesting clues (4). In Part B Maxwell in facts is confronted with the problem of connecting Weber's factor with the constants of a *continuous and fixed aether* and with the velocity *of the waves*, not with that of the particles, as in Weber's theory.

Maxwell's struggle to find arguments in support of his theory, is, in my opinion, significantly documented in his various attempts to connect Weber's factors to the constants of the aether, a necessary prerequisite in his theory to reach the conclusion that the same factor represented the velocity of electromagnetic waves in aether.

A brief review of Maxwell's attempts may be usefull before proceeding to a more detailed analysis.

In his (5) "On Physical Lines..." (1861–62) the connection between Weber's factor and the velocity of the electromagnetic waves turns out to be depen-

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(2) W. Weber, R. Kohlrausch, "Über die Elektricitätsmenge, welche bei galvanischen Strömen durch den Querschnitt der Kette fliesst" (1856), *Werke*, *quot.*, 597–608.

(3) Among them: J. Bromberg, "Maxwell's Displacement Current and his Theory of Light", *Arch. Hist. Exact Sci.*, *4*, 1967, 219–234; C. W. F. Everitt, *James Clerk Maxwell* (*Physicist and Natural Philosopher*), C. Scribner's Sons, New York 1975.

(4) S. D'Agostino, "Weber and Maxwell on the Discovery of the Velocity of light in 19th Century Electrodynamics", in M. D. Grmek, R. S. Cohen, G. Cimino (eds), *On Scientific Discovery*, Reidel, Dordrecht 1980, 281–293.

(5) J. C. Maxwell, "On Physical Lines of Force", *Phil. Mag.*, *21*; Part I, March 1861, Part II, April and May 1861; Part III, January and February 1862; also in *The Scientific Papers of James Clerk Maxwell*, W. D. Niven (ed.), 2 vols, Dover Reprint, vol. I, 451–513.

dent on an "ad hoc" hypotheses, the adoption of an hypothetical elastic model for aether. The wave velocity is also derived from the known velocity of an elastic wave.

In his (6) "A Dynamical Theory..." (1864) the novelty resides in deriving the electromagnetic waves velocity from a D'Alembert-type equation, and his method implies a certain degree of immunity from the elastic argument. The recourse to the theory of two Absolute System of Units, in connection with Maxwell's engagement in the works of a Committee for Electrical Standards (1863), was helpfull in relating Weber's factors with the aethereal constants.

In his (7) "On a Method..." (1868) an important result was reached: the correlation of Weber's factor with a ratio between electrostatic and electromagnetic forces.

In *A Treatise* (1873) Maxwell performs (8) a grandiose operation: the construction of a complete metrological theory of two Absolute Systems of units for electric and magnetic quantities, and the development of a consistent theory of dimensions of these quantities. The operation aimed to introduce consistently Weber's factor into the theory. In fact, as a consequence of this operation, Weber's factor appears consistently in every dimensional and numerical relations between the units of the quantities in the two different systems. The equality is then demonstrated by simple formal mathematical passages.

There is perhaps no better way to appreciate Maxwell's ingenuity than to follow in some detail, throught the above mentioned works, his attempts in overcoming the difficulties which were inherent in his various solutions to the problem of Weber's factor.

3. The elastic theories of optics and Maxwell's electromagnetic theory

To my knowledge, Maxwell, for the first time, quotes Weber in a letter (9) to Thomson, dated may 15, 1855; suggested by Thomson's Maxwell reads Weber's *Elektrodynamische Maasbestimmungen* and his comment is:

> I have been examining his (*i.e.* Weber's) mode of connecting electrostatics with electrodynamics, induction etc., I confess I like it not at first... but I suppose the rest of his view are founded on experiments which are trustworthy as well as elaborate.

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(6) J. C. Maxwell "A Dynamical Theory of the Electromagnetic Field", *Trans. Roy. Soc.*, *CLV*, received: October 27, read: December 8, 1864; also in *The Scientific...*, *quot.*, vol. 1, 527-597.

(7) J. C. Maxwell, "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light", *Phil. Trans.*, *48*, received June 10, read June 18, 1868; also in *The Scientific...*, *quot.*, vol. 2, 125-143.

(8) J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Unaltered reprint of 3th ed., 1891, Dover, New York 1954.

(9) J. Larmor (ed.), *The origins of Clerk Maxwell's Electrical Ideas, as Described in Familiar Letters to William Thomson*, Cambridge 1937, 705.

There is no mention, for the moment, of any idea of a theory of light. Maxwell mentions this theory in a letter (10) mailed from London, on December 10, 1861, in which he describes to Thomson his particular model of particles and celles adding that he " ... calculated the relation between the forces and the displacement on the supposition that the celles are spherical and that their cubic and linear elasticities are connected as in a " perfect " solid... ". It can be argued that Maxwell worked (11) at this model and the theory thereoff in Summer 1861. He then speaks of " Weber's value of the statical measure of a unit of electric current " from which he deduced the relation between the elasticity and density of the cells, and the velocity of transverse ondulations. The conclusion is " that the magnetic and luminiferous media are identical and that Weber's number is really, as it appears to be, one half the velocity of light (12) in millimeters per second ". Maxwell's original impact on Weber's metrology and the velocity of light was destined to a great " seguito ".

The British Association for the Advancement of Science, founded in 1831, appointed in 1862 a Committee on Electrical Standards, aiming at the determination of the best system of Electrical Units. C. Wheatstone, W. Thomson, F. Jenkin, J. C. Maxwell, C. W. Siemens, B. Stewart, J. P. Joule, were among the Committee members (13).

When Maxwell joined the Committee in 1862, the first two Parts of his 1861–62 " A Physical Lines " had been recently published. These Parts dealt mainly with a field theory of electrostatics, Faraday's induction and Ampere's forces. In Part 3, " The Theory of Molecular Vortices Applied to Statical Electricity ", Maxwell presents (14) his first version of the electromagnetic theory of light, and for the first time, in a published paper, he refers to the experimental work of W. Weber and R. Kohlrausch.

In order to fully grasp Maxwell's conception at this stage, one should consider that his impact on optics followed an half-a-century of researches and publications in elastic theories of an optical aether. Augustin Fresnel and Augustin Luois Chauchy, among others, succeeded to show how some complicated effects of christalline optics could be explained by apt elastic hypothesis and mathematical analysis. George Green and Gabriel Stockes in England had pursued the methods of mathematical-physics to works out elastic theories of an optical aether. Maxwell himself contributed (15) to elasticity in one of his first scientific papers.

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(10) *Ibidem*, 729.

(11) For the argument of my paper is irrilevant wether Maxwell's identification of the velocity of electromagnetic waves with that of light was or was not a discovery. On this point: J. BROMBERG, "Maxwell's...", *quot.*, 227; P. N. HEIMANN, "Maxwell and the Modes of Consistent Representation" *Arch. Hist. Exact Sci.*, *6*, 1970, 171–213: 193. Both authors agree that Maxwell did not know Weber's numerical value when he discovered that the velocity of his magnetic waves was equal to Weber's factor.

(12) Because of Weber's choice of electrodynamic units the factor 1/2 appeared before the velocity *c*. However Weber adopted also electromagnetic units (S. D'AGOSTINO, "Weber and Maxwell...", *quot.*, 285).

(13) See "Report of the Committee...", note 20, below.

(14) Note 22, below.

(15) J. C. MAXWELL, *The Scientific...*, *quot.*, 30.

This background is relevant for an assessment of Maxwell's initial approach to Weber's factor, explaining why he accepted that, in his first identification of this factor with the velocity of light, the elastic theory could have still a role. In fact in Part 3 of P.L. he presents (16) a detailed hypothesis on the elastic properties of an " electric aether ", through which the relation between the " electric displacement " and the " electric force " are deduced. Analogy plays an important role in the deduction: each quantity is endowed with a double referent, one to the elastic and the other to the electric theory. The quantity E, " a coefficient dependent on the nature of the dielectric " connects the " electromotive force " R to the " electric displacement " h, in the equation: $R = -4 E^2 h$. The same equation possesses however a counterpart in elasticity, representing Hooke's low of force, a strain–stress relation. Similary, the magnetic quantity μ, the " coefficient of magnetic induction ", it also possesses counterpart: the " density of the matter of the vortices ". Once the elastic roles of E^2 and μ are assigned, the relation $V = \sqrt{E^2/\mu}$ correctly represents the velocity of propagation of a wave in an elastic medium and, assuming that the " density of the matter of the vortices ", μ, is unitary, one has: $V = E$.

Maxwell has now to show that this velocity equals Weber's factor. From equation $R = -4 E^2 h$ and a theorem on the strain and energy in an elastic solid, he deduces (17) Coulomb's law, in the form: $F = - E^2 e_1 e_2 / r^2$ (second form). Because the units are, according to Maxwell, electromagnetic units, a comparison with the same law in the usual form, — *i.e.* " measured statically " — allows one to attribute to E the meaning of a conversion factor between " dynamic and electrostatic units ". Provided dynamic units are identified with the Weber's electromagnetic units, the factor above is then Weber's factor. In short, magnetic disturbances, predicted by theory, propagate with a velocity which is equal to Weber's factor and the velocity of light. Maxwell announces (18):

> I have deduced from this result the relation between the statical and the dynamical measures of electricity, and have shown, by a comparison of the electrodynamic experiments of M.M. Kohlrausch and Weber with the velocity of light as found by M. Fizeau, that the elasticity of the magnetic medium in air is the same of that of the luminiferous medium, if the two coextensive, and equally elastic media are not rather one medium.

The strenght of the argument is the partial analogy between electromagnetic phenomena, on one hand, and elastic behaviour of solids on the other, a rather well known and accepted analogy for the British electric sicentists of the middle of the century, ranging from Faraday's analogies on the behaviour of currents in cables, to the mathematical elaborations of elastic analogies of William Thomson in the fifties.

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(16) J. C. MAXWELL, "On Physical...", *quot.*, 495; J. BROMBERG, "Maxwell's...", *quot.*, 225.
(17) J. C. MAXWELL, *The Scientific...*, *quot.*, 498.
(18) *Ibidem*, 492.

In " On Physical Lines... " analogies play an intermediary role between electromagnetic-optical theories and elastic theories at two points: in the derivation of the law of propagation velocity and in the derivation of Coulomb's law in its second form. In the latter law the choice of the units is ambigous because there is not any precise reason as to why dynamic units are to be identified with Weber's electromagnetic units; the help of metrology is not yet a full one. In fact Maxwell's next step will be the introduction of two clearly defined Absolute Systems of units.

4. Metrology makes its entry on Maxwell's optical theory stage

We have a clear indication of Maxwell's engagement in the activities of the Committee on Electrical Standards in 1862-63. According (19) to one of Maxwell's students, W. D. Niven:

> In 1862-63 [Maxwell] took a prominent part in the experiments organized by a Committee of the British Association for the determination of electrical resistence in absolute measure and for placing electrical measurements on a satisfactory basis. In the experiments which were conducted at King's College upon a plan due to Sir W. Thomson, two long series of measurements were taken in successive years [..].

Maxwell and Thomson's activity had a determinant role in orienting the Committee's work towards a metrological programm of a high scientific level, inspired by Weber's Absolute Systems of electric and magnetic units. A perusal into the annual Reports of the British Association for the same period, documents (20) Maxwell and Kelvin's success in shifting the Committee program from the definition and construction of " etalons ", (a rather engineer-style approach to the problem of an intersubjective measurement of electric and magnetic units) to the apparently more difficult determination, by laborious laboratory measurements, of units in each of the two Absolute Systems.

In Maxwell's (and Fleming Jekin's) Appendix C to the " Report of the Committee... ", by the title " On the Elementary Relations between electric Measurements ", (1863), the conception of two complete Absolute Systems of electros-

(19) J. C. Maxwell, *The Scientific...*, *quot.*, Preface, XVI.

(20) " Report of the Committee appointed by the British Association on Standards of Electrical Resistence ", *The Report of the British Association for the Advancement of Science*, 1st series, 32, 1863, 111-176. C. W. F. Everitt highly evaluates this unduly neglected Maxwell's paper, because it " supplied a vital step " in the definition of a " dual system of electrical units " (C. W. F. Everitt, *James Clerk Maxwell...*, *quot.*, 100). I agree with Everitt on this point. He continues by stating that " by 1863, then, Maxwell had found a link of a purely phenomenological kind between electromagnetic quantities and the velocity of light" (*Ibidem*, 101). I think that the construction of the "dual sistem", from which Maxwell's new link was derived, is far from being phenomenological in its kind, although I admitt that the new link requires a minor number of ad hoc ipotheses in comparison with Physical Lines. For other aspects of the 1863 " Report ": S. D'Agostino " Experiment and theory in Maxwell's works", *Scientia*, *113*, 1978, 453-468.

tatic and electromagnetic units is presented (21) as the only one consistent with the present knowledge of electromagnetic phenomena and of their connection with the mechanical measurement of space, time and mass. It is emphasized that the completeness of the electrostatic System was made possible by the discovery of Joule's law on the thermal effect of an electric current in 1841.

It is noteworthy that Maxwell presents in this paper (22) various statements on his conception of " a quantity of electricity " clearly stating that " charge " for him is not necessarily distinct from matter as in Weber's conception:

> In speaking of a quantity of electricity, we need not conceive it as a separate think, or entity distinct from ponderable matter, any more than in speaking of sound we conceive it as having a distinct existence. Still it is convenient to speak of the intensity or velocity of sound, to avoid tedious circumlocution; and quite similarly we may speak of electricity, without for a moment immagining that any real electric fluid exists.

It is not a mere coincidence that in the above passage Maxwell neatly formulates his refusal to accept a Weber's type conception of charge (and consequently a convective conception of current), and that, in the same paper, he formulates for the first time a theory of units and dimensions, because, in my opinion (23), metrology and dimensions represent Maxwell's substitutes to Weber's conception.

In "On the Elementary Relation..., Maxwell presents (24) for the first time his theory of " dimensions " of physical quantities. What deserves our attention is the link he establishes between the dimension of a Quantity and its physical attributes, by assuming that if a quantity X has the dimension of a velocity, this quantity is a velocity. This link, once established, will make lowfull a transition from any dimensional to any physical property (transitional proposition for dimensions). One example concerns (25) the resistance R of

(21) "On the Elementary Relations between Electric Measurements", Appendix C to "Report of the Committee...", *quot.*, 130-163: 131.

(22) *Ibidem*, 136.

(23) In fact, Weber's conversion factor v is easily deduced from Weber's fundamental law, expressed as a difference between a Coulomb's force F_s and an Ampere's force between currents F_d:

$$F = F_s - F_d.$$

By the usual definition by Coulomb's law of an electric charge measured in electrostatic units, e_s, and in electrodynamic units (Ampere's force), e_d, Weber easily gets

$$e_s/e_d = v \; [v = \text{Weber's velocity}]$$

(W. Weber, R. Kohlrausch, "Über die Elektricitätsmenge...", *quot.*, 605). Because Maxwell rejected Weber's founfamental law and the inherent concept of charge and convective current, the only way left to him to introduce Weber's factor was that of exploiting dimensional relations in Coulomb's and Ampere's laws, thus:

$$[e_s] = [r][F_s][e_d] = [l][F_d][e_s/e_d] = [v]$$

[ ] Maxwell's symbol for dimension.

By the transitional property, Maxwell concludes: $e_s/e_d = v$. As regards definition of current, continuity equation and convective concept of current: E. Whittaker, *A History of the Theories of Aether and Electricity*, 2 vols., Nelson and Sons, London 1900-1953, vol. I, 176.

(24) "On the Elementary...", *quot.*, 132-135.

(25) "Report of the Committee...", *quot.*, 118.

a conducting wire. By Faraday's induction and Ohm's law, the resistence R can be expressed:

$$R = \sqrt{\frac{VSL}{C}} \qquad [8]$$

V, velocity of motion of the conductor of lenght L, trasversed by a current C in a magnetic field of intensity S.

> One curious consequence of these considerations is, that the resistance of a conductor in absolute measure is really expressed by a velocity; for, by equation (8), when $SL = C$ we have $R = V$, that is to say, the resistence of a conductor may be expressed or defined as equal to the velocity with which it must move, it placed in the conditions described, in order to generate a current equal to the product of the lenght of the conductor into the intensity of the magnetic field; or more simply, the resistence of a circuit *is* the velocity with which the conductor of unity lenght must move across a magnetic field of unit intensity in order to generate a unit current in the circuit (My underlining).

The transition from dimensional to physical property, introduced in the above-cited passage, will be emphasized and expanded in *A Treatise*. The Report for 1863 ends with the annuncement that a measurement of Weber's factor is included in the plans for the Committee future works.

In a *letter* (26) by Maxwell to Gabriel Stokes, dated October 15, 1864, we have a first-hand indication that Maxwell thought he found a way of by-passing his " On Physical Lines... " approach to the problem:

> I have now got materials for calculating the velocity of transmission of a magnetic disturbance throught air founded on experimental evidence, without any hypothesis about the structure of the medium or any mechanical explanation of electricity and magnetism.

In the same letter Maxwell mentions the problem of the velocity of " slow " and " rapid " disturbances and so concludes the passage: " We are divising methods to determine this velocity = electromagnetic: electrostatic units of electricity... "

He refers to his and Jenkin's experiment for a measurement of the " capacity of a conductor both ways " and to " a plan of a direct equilibrium between an electromagnetic repulsion and electrostatic attraction... "; this is a clear hint that the experiment which he is preparing is that described in his " A Note... " (1868). I think that in the passage above mentioned Maxwell meant by " experimental evidence " the more direct link between the measurements of Weber's factor and the velocity of the waves, that he had now achieved throught the metrology of the two systems.

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(26) J. Larmor (ed.), *Memoir and Scientific Correspondence of the Late Sir George Gabriel Stokes*, 2 vols., Cambridge 1907, vol. 2, 26, quoted in: J. Bromberg, " Maxwell's... ", *quot.*

5. The elimination of hypotheses and the experimental evidence in "A Dynamical Theory of the Electromagnetic Field"

This achievement is openly evidenced in his "A Dynamical Theory..." (1865). The electromagnetic theory of light is there presented in a larger generality, a consequence of the development of the metrology of the two systems. In "A Dynamical Theory..." Maxwell derives (27) the second form of Coulomb's law from a field-energy expression (a remnant of the elastic analogy) and from Gauss law:

$$[\text{Energy}] = \text{Pdf} = 1/2\ \text{Pf},\ \text{div f} = -e,\ \text{P} = \text{kf}$$

The burden of the proof that Coulomb's law is expressed in electromagnetic units rests on Gauss law, and the expression for energy. Because the theory of elasticity is still in the background, the theory is not a pure phenomenological one nor it is purely founded on metrological arguments.

However the recourse to elastic theory has become now hidden in the background. In fact, once the two Absolute Systems of Units are introduced in the theory, Coulomb's law in the second form is thus "expressed in terms of the Electromagnetic System of measurement which is founded on the mechanical action between currents":

$$\frac{k}{4\mu}\,\frac{e_1\ e_2}{r^2} = v^2\,\frac{e_1\ e_2}{r^2}$$

e_1, e_2, electric charges in electromagnetic units; k the "dielectric capacity of ether": $k = 4\pi v^2$.

By comparison (28) with the same law in electrostatic units: $n_1\ n_2/r^2$, v, in the equation above, is Weber's ratio of units. The other important achievement is the derivation (29), on purely electromagnetic ground, of a D'Alembert-type equation for the magnetic field, where k enters as a factor in the displacement-current term.

The propagation velocity is:

$$V = +\sqrt{\frac{k}{4\pi\mu}}$$

Since in electromagnetic units, in air, $\mu = 1$:

$$V = v$$

the propagation velocity of the waves equals Weber's velocity and conversion factor.

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(27) J. C. Maxwell, "A Dynamical...", *quot.*, 568, 536.
(28) *Ibidem*, 569.
(29) *Ibidem*, 579.

The mechano-elastic analogies are now (30), according to Maxwell, relegated to the role of illustrations to assist the reader's immagination:

> In using such words as electric momentum and electric elasticity in reference to the known phenomena of the induction of currents and the polarization of dielectrics, I wish merely to direct the mind of the reader to mechanical phenomena which will assist him in understanding the electric ones. All such phrases in the present papers are to be considered as illustrative, not as explanatory.

" A Dynamical Theory... " ends (31) with a comparison between the self-induction coefficient value of a coil, calculated through Maxwell's new field theory, and that measured by the Committee's experiment at King's College, London, 1863: $L = 430165$ metres (calculated), $L = 456748$ metres (averadged by method of least squares). Another evidence that Maxwell was also inspired by problems raised by his work with the Committee.

6. Velocity of light and comparison of forces in 1868

The Weber's factor measurement, announced as part of the Committee's programm in the 1863 Report, was delaied untill 1868. A short summary of two methods for the same experiment, by William Thomson and Maxwell himself, was published in the Report of the Association for 1869. Besides, Maxwell presented a Memoir to the Royal Society, published (32) in the *Phylosophical Transactions*, in June 1868 with the title: " On a Method of Making a Direct Comparison of Electrostatic and Electromagnetic Force: With A Note on the Electromagnetic Theory of Light ".

The first part of this Memoir deals with a description of Maxwell's experiment, which was cleverly conceived as a balance of an electrostatic attractive force, (between two metal disks connected to two different points of a conducting wire), and a magnetic repulsive force between two coils of the same wire, fixed to the same disks.

The two forces were expressed respectively in terms of Coulomb and Ampère's laws and Weber's factor was introduced (33) in the balance equation as a conversion factor between the electrostatic units (Coulomb's law) and electromagnetic units (Ampère's law). Note that two systems of units were used consistently in the same equations (the conversion factor was introduced to satisfy an homogeneity principle); this Gauss-type method will be abandoned in A

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(30) *Ibidem*, 563-564.
(31) *Ibidem*, 589-597.
(32) J. C. Maxwell "On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light", *Phil. Trans., Roy. Soc. of London*, *158*, 1868, 643-658; also in J. C. Maxwell, *The Scientific..., quot.*, vol. 2, 125-143.
(33) *Ibidem*, 128.

Treatise favouring the use of one system at a time. Due to the fact that eectrostatic force was produced by a potential gradient across a high resistance (28798 OHMS) and that in the electromagnetic system a resistance has the dimensions of a velocity, Weber's factor was measured (34) either in terms of this resistance or in "metres (sic) per second":

$$v = 288\,000\,000 \text{ metres per second}$$

(Everadge on eleven results. the "probable error" is, according to Maxwell, "about one-sixth per cent"). Maxwell thus measures with his own hands Weber's factor and he considers the result a good approximation to Weber and Kohlrausch's v = 310 740 000 metres per second.

It is relevant for my thesis for the importance of the deduction of Weber's equality in Maxwell's theory that, in the theoretical Part of his Memoir, Maxwell presents (35) this deduction alone as a proof for his electromagnetic theory of light.

The statement of the electromagnetic theory of light in my former paper [Maxwell refers to his 1865 "Dynamical Theory..."] was connected with several other electromagnetic investigations, and was therefore not easily understood when taken by itself. I propose, therefore, to state it in what I think the simplest form, deducing it from admitted facts, and showing the connection between the experiments already described [Maxwell refers to his measure of Weber's factor] and those which determine the velocity of light.

The Scottish physicists supports his field theory by showing that both rival Riemann and Lorenz's theories of retarded action, lead to paradoxes.

In the "Note..." (36), an application of the field formulations of Faraday's induction law and Ampère's law (with the displacement current term), gives a propagation equation for a magnetic field (D'Alembert type) and a propagation velocity V in the same form as in "Dynamical Theory..." The novelty consists in that the balance equation for the equilibrium of forces is rewritten, but, this time, in terms of fields and it is compared to the former equation for the balance of forces, where Weber's factor appears as a conversion for units.

The comparison gives $v^2 = \mu k/4\pi$, *i.e.* Weber's factor in terms of k and μ. Equating it with the above (1865) expression for V, the conclusion (37) is:

$$\mu V = v$$

Adding the statement that, in air, μ is "*assumed* equal to unity" (underlined in the original): $V = v$.

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(34) *Ibidem*, 134-135.
(35) *Ibidem*, 138.
(36) *Ibidem*, 140-141.
(37) J. C. Maxwell, "On a Method...", *quot.*, 143.

Thus Weber's velocity becames the velocity of light!

In the above passage, Maxwell ranks his proof to no less then a deduction "from admitted facts", an homage to Newton's unperishable pronouncement. The procedure has the advantage to avoid the Coulomb's law derivation in the second form from elastic analogies, but it uses ambiguous metrological considerations in "assuming" $\mu = 1$, in air and in *every* system. The assimilation of Weber's factor to a ratio between electrostatic and electromagnetic forces, brings into Maxwell's theory features that are very akin to Weber's ones. However, Maxwell's deduction lacks the generality of Weber's law of force, from which the same ratio derives its full theoretical justification. Last but not the least, Maxwell abandoned this approach in *A Treatise*, in favour of another one founded on a complete theory of the Absolute Systems and a theory of dimensions.

7. Metrology and theory of dimensions in *A Treatise of Electricity and Magnetism*

In *A Treatise on Electricity and Magnetism* (1st edition 1873), Maxwell refers (38) to Fechner and Weber's convective conception of electric current as "an exceedingly artificial hypothesis" which he does not accept because of its inconsistency with energy conservation and because "electricity in the wire cannot be considered as the moving body in which we are to find this energy, for the energy of a moving body does not depend on anything external to itself, whereas the presence of other bodies near the current alters its energy". I think that this passage and Maxwell's 1863 statement (quoted above) give sufficient evidence to prove that he does not accept Weber's idea that a current is is a motion of charges (39).

In the same work he gives a stronger emphasis to the theory of the completeness of two Absolute Systems of units and the related theory of Dymensions. These theories are presented (40) in the introductory Chapter, "Preliminary", and in Part 4, "Electromagnetism", particularly Chapter 10, "Dimensions of Electric Units".

He remarks that the two systems are non consistent, as far as dimensions are concerned — in the sense that the same quantity has different dimensions in the two systems — and he proposes: "to begin by stating those relations between the different units which are common to both systems". For instance the

(38) J. C. Maxwell, *A Treatise..., quot.*, Art. 552. See also for the same problem: Art. 12, Art. 230.

(39) Concerning Maxwell's view on the nature of an electric current, Everitt (*James Clerk Maxwell..., quot.*, 127) expresses the same opinion: "Maxwell's current is not the motion of charge, but the motion of a continuous *uncharged* quantity (not necessarily a substance); his charge is the measure of the displacement of that quantity relative to space". However Everitt, in the same passage, cautions the reader "that other opinions are possible".

(40) J. C. Maxwell, *A Treatise..., quot.*, Arts. 620-629.

following products have (41) always (i.e. in both systems) the dimension of energy:

quantity of electricity × electric potential
quantity of free magnetism × magnetic potential
electrokinetic momentum × electric current.

Other products have dimension of an energy-density.

Then he presents (42) a symmetric arrangement of quantities in two lines, in such an order that " the quantities in the first line are derived by e [the electric charge] by the same operations as the corresponding quantities in the second line are derived from *m* [the magnetic charge]. All the relations given above are true wathever system of units we adopt ". The reason as to why Maxwell underlines in *A Treatise* the dimensional invariance of certain operations, is that he wants to prepare the groud in view of his approach to Weber's problem. The following paragraph (43) states:

The only systems of any scientific value are the electrostatic and the electromagnetic system.

In *A Treatise* the electrostatic and electromagnetic system are explicity founded on Coulomb's laws for " quantities of electricity " and " strenght of magnetic pole ", respectively (44).

I think that Maxwell's restiction to the two systems of the variety of conventionally conceivable systems of units — a restriction which is foreign to Weber's metrological theories — would be for us nuexplainable if we lack an adequate understanding of what I have called the " transitional property of dimensions ".

In fact in Maxwell's following pages are listed a " Table of Dimensions " of the electromagnetic quantities (45) and a table of their ratios (46), which "are in certain cases of scientific importance ". Among these ratios the " specific inductive capacity of a dielectric *K* " — related to the former *k* by $K = 1/k$ — has null dimension in the electrostatic system, and the inverse-square-of-a-velocity dimensions in the elctromagnetic system; the " magnetic inductive capacity " μ has (47) the reciprocal properties.

The following property of the ratio between units of a quantity of electricity in the two systems is then (48) announced:

If the units of length, mass, and time are the same in the two systems, the number of electrostatic units of electricity contained in one electromagnetic unit

(41) *Ibidem*, Art. 622.
(42) *Ibidem*, Art. 624.
(43) *Ibidem*, Art. 625.
(44) *Ibidem*, Art. 625.
(45) *Ibidem*, Art. 626.
(46) *Ibidem*, Art. 627.
(47) *Ibidem*, Art. 627.
(48) *Ibidem*, Art. 628.

is numerically equal to a certain velocity, the absolute value of which does not depend on the magnitude of the foundamental units employed. This velocity is an important physical quantity, which we shall denote by the symbol v.

This question is taken up again in Chapter 19, " Comparison of the Electrostatic which the Electromagnetic Units " (49).

It appears from the table of dimensions, Art. 628, that the number of electrostatic units of electricity in one electromagnetic unit varies inversely as the magnitude of the unit of lenght, and directly as the magnitude of the unit of the time we adopt.

It therefore we determine a velocity which is represented numerically by this number, then, even if we adopt new units of lenght and time, the number representing this velocity will still be the number of electrostatic units of electricity in one electromagnetic unit, according to the new system of measurement.

The dimensional ratio between the two units of quantity of electricity (*i.e.* charge) in the two systems in thus assimilated to a physical quantity, a velocity. The transition is then from dimensional to physical property: if a quantity X has the dimension of a velocity, X *is* (*i.e.* physically represents) a velocity (Transitional proposition).

This last argument, based on metrological and dimensional considerations, should have not suited entirely Maxwell, if he immediatly afterwords (50), in the same Chapter, turns to another type of demostration. " To show that the quantity we are in search of is really a velocity " and " to obtain a physical conception of this velocity ". This proof resembles closely a deduction from Fundamental Kraft — *à la* Weber — if it were not for the notable difference that it deals with macroscopic forces between charged macroscopic bodies in motion. The result of this demonstration is that the velocity in question is a relative velocity of motion between charged bodies, not a propagation velocity of waves, the true object of Maxwell's present theory!

After all that, in Chapter XX "Electromagnetic Theory of Light" (51) the task of connecting this velocity with the aetherial constant is now (52) facilitated by the theorems on dimensions (and the implicit acceptance of the transitional proposition):

The quantity V, in Art. 784, which expresses the velocity of propagation of electromagnetic disturbances in a non-conducting medium is, by equation (10), equal to $1/\sqrt{K\mu}$.

If the medium is air, and if we adopt the electrostatic system of measurement, $K = 1$ and $\mu = 1/v^2$, so that $V = v$, or the velocity of propagation is numerically equal to the number of electrostatic units of electricity in one electromagnetic unit. If we adopt the electromagnetic system, $K = 1/v^2$ and $\mu = 1$, so that the equation $V = v$ is still true.

(49) *Ibidem*, Art. 768.
(50) *Ibidem*, Art. 768; S. D'Agostino, " Experiment and Theory... ", *quot.*
(51) J. C. Maxwell, *A Treatise...*, *quot.*, Arts. 781-805.
(52) *Ibidem*, Art. 786.

On the theory that light is an electromagnetic disturbance propagated in the same medium through which other electromagnetic actions are transmitted, V must be the velocity of light, a quantity the value of which has been estimated by several methods. On the other hand, v is the number of electrostatic units of electricity in one electromagnetic unit, and the methods of determining this quantity habe been described in the last chapter. They are quite independent of the methods of finding the velocity of light. Hence the agreement or disagreement of the values of V and of v furnishes a test of the electromagnetic theory of light.

The demonstration of Part B has been reduced to purely formal passages!

8. Dimensions, electromagnetic units and symbols in physics: an historical survey

Maxwell's metrological theories can be better understood when situated in their historical background, the development of the concepts of systems of units and of dimensions in 19th century physics. Although it is to Weber that we owe the introduction (53) of a plurality of sistems of units in electrodynamics, the discovery of a constant c, and its measurement as a ratio of units, however the german scientist does not deal explicitly with dimensions in his work in electrodynamics. His approach to electrodynamics, differently from Maxwell, was supported by a convective conception of currents and by the related fundamental force (54).

Joseph Fourier is commonly considered the first who explicitly introduced dimensional arguments into his *Theorie Analitique*, and Maxwell quoted him as his source in his 1863 Report and in *A Treatise*. Invariace of the equations under change of units is Fourier's main concern in introducting (55) dimensions:

> Il faut maintenant remarquer que chaque gradeur indéterminée ou costante a une dimension qui lui est prope et que les terms d'une même équation ne pourraient pas être comparés, s'ils n'avaient point le même exposant de dimension. Nous avons introduit cette considération dans la Théorie de la chaleur pour rendre nos définitions plus fixes et servir à vérifier le calcul; elle dérive des notions primordiales sur les quantités: c'est pour cette raison que, dans la Geometrie et dans la Mecanique, elle équivaut aux lemmes fondamentaux que ls Grecs nous ont laissés sans demonstration...
>
> Dans la théorie analytique de la chaleur, toute equation (E) exprime une relation nécessaire entre des grandeurs subsistantes x, t, v, c, h, k. Cette relation ne dépend point du choix de l'unité de longuer, qui de sa nature est contingent; c'est-à-dire que, si l'on prenait une unité différent pour measurer les dimensions linéaires, l'equation (E) serait encore la même.

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(53) W. Weber, *Elek. Maasbest insb. Zurückfuer. der Stromint. Mess. auf Mech. Maass*, in W. Weber, *Werke*, *quot.*, 652.

(54) Notes 1, 2, 23.

(55) J. Fourier, *Theorie Analitique de la Chaleur*, Paris 1822, Art. 161.

In Fourier all the quantities in the equations have dimensions only with respect to length, time, temperature.

Maxwell for the first time run explicitly into dimensions in connection with his Absolute Systems, in Appendix C to the 1863 "Report" (56):

4. *Dimensions of Derived Units.* – Every measurement of which we have to speak involves as factors measurement of space, time and mass only; but these measurements enters sometime at one power, and sometimes at another. In passing from one set of foundamental units to another, and for other purposes, it is usefull to know at what power each of these fundamental measurements enters into the derived measures.

Thus the value of a force is directly proportional to a length and mass, but inversely proportional to the square of a time. This is expressed by saying that the *dimensions* of a force are LM/T [...].

The usage of capital letters in square brackets to indicate dimensions was introduced by Maxwell in *A Treatise*, and, in the same work, symbols in the equations explicitly represent quantities (*i.e.* a number "times" a unit) not numbers.

William Thomson is in good company with Maxwell in identifying (57) dimensions of a quantity in a given system with the quantity itself (*i.e.* Thomson accepted Maxwell's transitional proposition):

I suppose almost everyone present would think it simple idiocy if I went to say that the weight of that piece of chalk is the fourth power of seven or eight yards for hour; yet it would be perfectly good sense.

We have seen that this type of identification is what Maxwell proposes in his 1863 "Report" and in *A Treatise*. Some Maxwellians (*e.g.* J. E. H. Gordon, Everett etc.) followed Maxwell's views. William Kingdom Clifford, on the other side, took a more articulated position in 1878, underlying the conventionaly of the new symbolization of dimensions, justified by its convenience in the calculation of the change of units; he stressed (58) that this convenience could be however a cause for "non sense", if the meaning of dimensions is unduly extended:

[...] [V] = [L]/[T][...] Here the word *per* has been replaced by the sign for divided by. Now it is nonsense to say that a unit of velocity is a unit of length devided by a unit of time in the ordinary sense of the words. But we find it convenient

(56) "On the Elementary...", *quot.*, 132.

(57) Lord Kelvin, *Popular Lectures and Addresses*, vol. 1, London 1981 104, quoted by: A. O' Rahilly, *Electromagnetic Theory*, 2 vols., Dover reprint, New York 1965, vol. 2, 697.

(58) W. K. Clifford, *Elements of Dynamic*, 1, 1878, 49. Quoted by A. O'Rahilly, *Electromagnetic...*, *quot.*, vol. 2, 685. Also: M. J. Crowe, *A History of Vector Analysis*, University of Notre Dame Press, Notre Dame, 1967. Rahilly's opinion on dimensional quantities in physics is unambigously expressed (A. O'Rahilly, *Electromagnetic...*, *quot.*, vol. 2, 698): "The mystification sets in when we begin to misinterpret these numbers as complex qualitative happenings miracoulously susceptible to arithmetical operations such as raising to the forth power. It is precisely the failure to recognise the symbols of physics as ordinary numbers, which has led electricians into such a quagmire of futile and meaningless methaphysics".

to give a new meaning to the words " divided by " and to the symbols which shortly expresses them [..] this convenience is made manifest when we have to change from one unit to another [..].

Max Planck considers Maxwell's two Absolute Systems as part of a set of other possible systems. Planck's admission of a plurality of systems implies his conviction that the concept of dimension is relative to a given System and that it is " non sense " to search for the " real " dimension of a quantity. This conclusion (59) wipe out a possible objection to the plurality of dimensions of a quantity but, at the same time, deprives Maxwell's two systems theory of its label of being " the only scientific " theory and, dismisses Maxwell's argument in favour of Weber's equality:

> The fact that when a definitive physical quantity is measured in two different systems of units it has not only different numerical values, but also different dimensions, has often been interpreted as an inconsistency that demands explanation, and has given raise to the question of the *real* dimensions of a physical quantity. After the above discussion it is clear that the question has no more sense then inquiring into the *real* name of an object.

Giovanni Giorgi (1871–1950) was convinced in 1901 that the shortcomings of Maxwell's two Absolute Systems, as regards an appropriate size of their units for the practice, were not repairable through an appropriate re–definitions of the fundamental units, but required the introduction of a fourth fundamental unit. He urged to abandon the three unit system, which had been inspired, according to him, by Gauss and Weber's mechanicistic approach to electrodynamics. Giorgi proposed (60) the well known four-unit system, using the Ohm as the fourth unit for electric resistence. What he objected (61) in Maxwell's theory was also that the constants, in Coulomb's laws for electric and magnetic charges, could be equalized to unity, because they were not numerical coefficients but " physical constants ", an index of " the capacity of space of being charged with energy " (*i.e.* the three-unit system is overdetermined).

In the 1933 the American Committee of Physicists and Electricians submitted to the Special Committee of the Electrotechnical Commission the adoption of Giorgi four-unit system and the Commission in October 1933 approved the proposal.

In 1935 Arnold Sommerfeld decided (62) in favour of a four-unit system:

> The ortodox number three, which is at the base of the so–called absolute system of measurement, could be considered as mandatory as long as one

(59) M. Planck, *Theory of Electricity and Magnetism*, vol. III of M. Planck, *Introduction to Theoretical Physics*, Mcmillan, London 1932, first edition 1922.
(60) M. Loria, " Giovanni Giorgi ", in G. C. Gillispie (ed.), *Dictionary of Scientific Biography*, C. Scribner's Sons, New York 1972, vol. 5, 407.
(61) G. Giorgi, " Unità (sistemi di) ", *Enciclopedia Italiana*, Rizzoli 1937, 36 vols., vol. 34, 714–718, 716.
(62) A. Sommerfeld, " Über die Dimensionen der electrodynamische Grossen ", *Phys. Z.*, *35*, 1935, 814–820 (the passage was translated in english by the present author).

hoped to derive electricity from mechanics. This time is now over. One exerces a violence against electromagnetic quantities when one forces them, in the Procuste's bed of the three units; it can be shown that they are at ease in the four-unit system.

In his celebrated *Electrodynamics* (63), Sommerfeld warns against what he considers the funest pedagogical effects of Maxwell's two Absolute Systems: "We have frightened generations of students with these two sets of values for charges and field strenght [...]."
He also quotes among " so many other clarifications in the question of units ", a funny example by J. Wallott, in which a fundamental velocity is made to appear in acustics through an appropiate choise of constants and units.

Coming to a modern view on the problem of fundamental units and dimensions, Wolfgang Panofsky affirms (64) that " in classical mechanics conventional choices of appropriate systems of units present no problem ", however, as regards the electromagnetic theory " the conventions are of more recent origin and appear more controversial ". The fact that in this theory a fundamental constant, the velocity of light in vacuum c, appears, has the consequence that " physical laws scale correctly over arbitrary magnitudes only if ratios of length and time are held constant [...]. If the MLT system is used in the mechanical quantities in electromagnetic theory, the constant c having dimensions L/T will appear explicitly ".

It seems that the avove solution does not satisfy D.C. Ipsen (65), because " the idea that correspondent concepts (*i.e.* correspondent quantities) in the electrostatic and electromagnetic systems are different (have different dimensions) is rather unpleasant ".

9. Conclusions

There is today agreement among Maxwell's scholars that the innovations he brought into his electromagnetic theory, and into physics in general, also concern new mathematical (analytical and geometrical) properties of physical quantities. Analogical similarities, vectorial and tensorial symmetries of the electric and magnetic fields are the tools of Maxwell's handwork. They have an important role in the context of his electromagnetic theory.

What I think is noteworthy is that, in some instances, Maxwell invended (66) new symbols for physical quantities which significantly embed and express

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(63) A. Sommerfeld, *Electrodynamics*, (*Lectures on Theoretical Physics*), vol. III, Academic Press Boston Mass. 1964.
(64) W. Panofsky, *Classical Electricity and Magnetism*, Addyson Wesley, Boston 1955, 375.
(65) D. C. Ipsen, *Units. Dimensions and dimensionals Numbers*, Mc Graw-Hill, New York 1960, 91.
(66) M. J. Crowe, *A. History...*, *quot.*, 131 ff.

the afore mentioned mathematical features. I mean the notations (67) for the vectorial operators of curl, convergence, concentration, etc. some of which are Maxwell's originals (other are Kelvin's and Hamilton's). Among these novelties I think one should include Maxwell's original notation for the dimensions of physical quantities.

It is not a mere chance that, in the same period Kelvin and Hamilton, along with Maxwell, introduced new symbols in the mathematical codification of physical law; these scientists shared the implicit convinction that it proved unnecessarily restrictive of the potentiality of mathematics in physical theory to consider physical symbols as purely algebraic numbers.

I think it is in this train of ideas that Maxwell discovered that dimensional rules, besides being a usefull tool (in a given system of units) for checking the correctness of the passages in the equations, also possess other properties like invariance under the change of system etc. (he wrote Chapt. X of Part III of his *A Treatise* to examine these extra properties of dimensions). In so doing he overloaded dimensions with properties — such as the transitional property — which proved an unduly extension, or a streching, of the theory, and he attributed to his two absolute systems singularities which were considered, by some of his successors, restrictive of the free choice which is the consequence of the conventionality of any specific system.

Hidden inconsistencies are in fact found in the trensitional property. The very fact that the same physical quantity, for most of the electromagnetic quantities, has different dimensions in Maxwell's two systems (*i.e.* the transition is dependable on a particular system) casts into doub the general validity of the transition i.e. the reliability of dimensions as a source (counter part) of physical properties. As shown above, dimensions are however invariant in the two systems in the special case of V, the theory's true concern at this point. Maxwell's understatement of these inconsistencies and his acceptance of the transitional property is, in my opinion, the manifestation of an extreme tension in his logic of research in overcoming the formidable difficulty of Part B of his electromagnetic theory of light. This difficulty was mainly a consequence of his rejection of the convective conception of an electric current. His implicit recourse to what I called a transitional property was a way of circumventing it.

In my opinion Maxwell's stretching of his dimensional theory is also part of his idea of " completeness ". This was, so to speak, the positive side of the medal. The trend to the generalization of a theory towords its utmost general consequences, the " idea of completeness ", is by Siegel rightly considered (68) as one of the most intresting feature of Maxwell's style.

(67) *Ibidem*, 135.

(68) D. SIEGEL, "Completeness as a Goal in Maxwell's Electromagnetic Theory", *Isis*, *66*, 1975, 361-368.

I am conforted in this interpretation by Northon Wise's remark (69) which explicitly referes to Maxwell's dimensional analysis and its connection with the "completeness": "The idea of completeness, implicit in Maxwell's technique rather than in his methodology, also explains Maxwell's well known concern with dimensional analysis in physics and with the mathematical classification of physical quantities".

We should agree that Maxwell's idea of assigning new mathematical properties to physical quantities — and to the symbols which represent them in the equations — was immensely successfull; it seems then reasonable that he attempted an extension of dimensional properties to reach a demonstration that the rest of his theory corroborated with its valid results. Metrological and dimensional theories are thus consistent with the rest of Maxwell's conceptions and, as such, they should be accepted as an integrant part of Maxwell's paradigma, which includes also the important feature of the refusal of the convective conception of currents.

The later development of the theory of the electromagnetic field at the end of the century were in part devoted to eliminate these inconsistencies. Antoon Lorentz, introduced into the theory the convective conception, side by side with the field equations, thus vindicating, in a way, Weber's convective idea. As it is well known, Maxwell's equations have a fundamental role in the Relativity Theory, but Weber's ratio can also be deduced (70) from this theory, in a first order approwimations, as a consequence of Lorentz's transformations. Then it can be argued that Einstein's Special Relativity, includes part not only of Maxwell's theories but also of Weber's rival ones.

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(69) N. Wise "The Maxwell Literature and British Dynamical Theory", *Hist. Stud. Phys. Sci.*, *13*, 1981, 175-205; 195.

(70) R. Feynman, L. Leighton, E. Sands, *The Feynman Lectures on Physics*, 3 vols., Addyson Wesley, Boston 1963, vol. II, 26-29.