

Formule per il calcolo numerico
degli integrali coulombiani con orbitali, di tipo
 s, p, d ed f , con n intero e non intero (*). Nota II.

Gli integrali coulombiani sono indubbiamente i più importanti fra gli integrali d'interazione elettronica bicentrici: sia in quanto numericamente più grandi rispetto a quelli ibridi e di scambio, sia perché sussistono teorie, come quella di PARKER e PAIR, basate essenzialmente sulla sola conoscenza di questi integrali.

Essi hanno perciò costituito argomento di studio da parte di numerosi Ricercatori (1).

La conoscenza di questi integrali rappresenta senza dubbio una prima tappa per l'ulteriore sviluppo di un moderno calcolo della struttura elettronica molecolare.

I tipi di integrali coulombiani studiati dai precedenti Autori si riferiscono essenzialmente alle funzioni atomiche di SLATER relative ad orbitali di tipo s, p, e , in qualche caso più raro, anche ad orbitali di tipo d , sempre però caratterizzati da numero quantico n intero. Mediante questi integrali già noti si possono studiare, in prima approssimazione, le molecole formate da atomi del primo e secondo periodo, nelle quali le funzioni d'onda di SLATER, con n intero, costituiscono approssimazioni abbastanza buone dei migliori orbitali atomici numerici di HARTREE. Nel caso invece di atomi a più alto numero atomico, questa approssimazione diventa sempre meno soddisfacente, ed è necessario perciò ricorrere, nell'approssimazione di n intero, a combinazioni lineari di più funzioni d'onda, con evidente complicazione dei calcoli e perdita di significato fisico dell'orbitale usato. Ci è sembrato pertanto utile impostare il calcolo degli integrali bicentrici coulombiani relativi a funzioni di SLATER, caratterizzate da qualsiasi valore di n , intero e non intero.

Abbiamo inoltre esteso i calcoli fino agli orbitali « f ».

Questa generalizzazione sui valori di n , necessaria come già si è detto per gli orbitali degli atomi più pesanti, è stata da noi estesa anche al caso di quelli più leg-

(*) Memoria presentata dall'Accademico GIOVANNI BATTISTA BONINO.

geri, dove, pur essendo meno necessaria, può portare tuttavia ad un miglioramento dei risultati.

Dal punto di vista del calcolo, la considerazione di numeri n non interi presenta notevoli difficoltà: mentre infatti con n interi è possibile uno sviluppo analitico dei calcoli (sia pure servendosi dello sviluppo in serie di $1/r_{1,2}$), un analogo sviluppo analitico completo non è ulteriormente applicabile al caso di orbitali con n non intero.

Come già notato in un precedente lavoro (*) è stato perciò necessario impostare il problema secondo i metodi del calcolo numerico. Questa ultima via è resa possibile mediante l'uso dei moderni calcolatori elettronici.

Il problema più grave che si è presentato è stato quello di scegliere una opportuna suddivisione dei campi di integrazione: può infatti capitare che i calcoli comportino differenze fra numeri che diventano sempre più grandi, in modo che la precisione va quasi completamente persa, fino ad ottenere risultati privi di significato.

Questa difficoltà generalmente si elimina, o impostando i calcoli in doppia precisione con un assai grande aumento dei tempi di calcolo, o meglio, con un più opportuno studio dei campi e delle variabili di integrazione.

In questo lavoro abbiamo essenzialmente studiato il problema dal secondo punto di vista, sviluppando i calcoli, analiticamente, campo per campo, per quanto possibile, in modo da giungere a formule adatte al calcolo numerico, senza dover ricorrere al metodo della doppia precisione.

Il criterio adottato nella suddivisione dei campi di integrazione è basato sull'analisi del comportamento della funzione integranda, in modo ch'essa abbia uno stesso andamento nello stesso campo.

Le funzioni di base scelte sono le funzioni reali del tipo di Slater date da :

$$\sigma^{n,l,m} = \left[\frac{(2\zeta)^{2n+1}}{(2n)!} \right]^{1/2} r^{n-1} e^{-\zeta r} S_{l,m}(\vartheta, \varphi) \quad (1)$$

dove

$$\begin{cases} S_{l,0} = Y_l^0 \\ S_{l,|m|} = \frac{1}{\sqrt{2}} (Y_l^{-|m|} + Y_l^{|m|}) \\ S_{l,-|m|} = \frac{i}{\sqrt{2}} (Y_l^{-|m|} - Y_l^{|m|}) \end{cases} \quad (2)$$

n è un numero qualsiasi positivo, ζ è la carica efficace, caratteristica dell'atomo e dell'orbitale, mentre le Y_l^m sono le funzioni armoniche sferiche definite da :

$$Y_l^m = \sqrt{\frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!}} \text{sen}^{|m|} \vartheta P_l^{|m|}(\cos \vartheta) e^{im\varphi} \quad (3)$$

dove $P_l^{|m|}$ indica il polinomio associato di LEGENDRE di grado l e ordine $|m|$.

Gli orbitali con $l=0, 1, 2, 3$ sono stati indicati, come nella annotazione corrente, con lettere s, p, d, f , seguite da un indice che è lo stesso (in m) della $S_{l,|m|}$.

Questa notazione è stata ritenuta più comoda soprattutto per gli orbitali f , di quella che usa le lettere x, y, z . Il confronto fra le due notazioni è riportato per comodità nella tabella seguente:

| | |
|--------------------------|-------------|
| $x = x$ | $S_{30,0}$ |
| $p_x = p_x$ | $S_{31,0}$ |
| $p_1 = p_x$ | $S_{31,1}$ |
| $p_{-1} = p_y$ | $S_{31,-1}$ |
| $d_{xy} = d_{xy}$ | $S_{32,0}$ |
| $d_{z^2} = d_{z^2}$ | $S_{32,1}$ |
| $d_{-1} = d_{yz}$ | $S_{32,-1}$ |
| $d_{z^2} = d_{x^2-y^2}$ | $S_{32,2}$ |
| $d_{-2} = d_{xy}$ | $S_{32,-2}$ |
| $f_0 = f_0(x^2-3z^2)$ | $S_{33,0}$ |
| $f_1 = f_x(x^2-1)$ | $S_{33,1}$ |
| $f_{-1} = f_y(x^2-1)$ | $S_{33,-1}$ |
| $f_2 = f_z(x^2-3z^2)$ | $S_{33,2}$ |
| $f_{-2} = f_{xy}$ | $S_{33,-2}$ |
| $f_3 = f_x(x^2-3z^2)$ | $S_{33,3}$ |
| $f_{-3} = f_y(x^2-3z^2)$ | $S_{33,-3}$ |

Gli assi di riferimento sono stati orientati come in fig. 1.

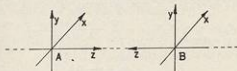


Fig. 1

Un generico integrale coulombiano, costruito con 4 funzioni del tipo (1) è caratterizzato da 4 gruppi di indici, $n_1, l_1, m_1; n_2, l_2, m_2; n_3, l_3, m_3; n_4, l_4, m_4$; e può scriversi:

$$I = \int \omega_a(1) \frac{1}{r_{12}} \omega_b(2) d\tau_1 d\tau_2 \quad (4)$$

dove gli indici a e b indicano i due centri rispetto ai quali sono riferiti gli orbitali, i numeri 1 e 2 stanno rispettivamente per le tre coordinate dei due elettroni. Le due distribuzioni di carica ω_a ed ω_b , esplicitate, sono:

$$\begin{aligned} \omega_a(1) &= \left[\frac{(2 l_1)! (2 l_2)! (2 l_3)! (2 l_4)!}{(2 n_1)! (2 n_2)!} \right]^{1/2} r_1^{n_1+n_2-2} e^{-(n_1+n_2)} r_2 S_{l_1, m_1}(\vartheta_{1a}, \varphi_1) S_{l_2, m_2}(\vartheta_{1a}, \varphi_1) \\ \omega_b(2) &= \left[\frac{(2 l_3)! (2 l_4)! (2 l_1)! (2 l_2)!}{(2 n_3)! (2 n_4)!} \right]^{1/2} r_3^{n_3+n_4-2} e^{-(n_3+n_4)} r_4 S_{l_3, m_3}(\vartheta_{2b}, \varphi_2) S_{l_4, m_4}(\vartheta_{2b}, \varphi_2) \end{aligned} \quad (5)$$

In ciascuna di queste due distribuzioni la parte angolare è data dal prodotto di due funzioni $S_{l,m}$ del tipo (2), e può essere sempre espressa come una combinazione

lineare delle stesse funzioni. Per trovare tali combinazioni possiamo partire dalla (2) e sfruttare la formula

$$Y_{l_1}^{m_1} Y_{l_2}^{m_2} = \sum_L \left[\frac{(2l_1 + 1)(2l_2 + 1)}{4\pi(2L + 1)} \right]^{1/2} C(l_1 l_2 L; 0 0) C(l_1 l_2 L; m_1 m_2) Y_L^{m_1 + m_2} \quad (6)$$

I coefficienti $C(l_1 l_2 L; 0 0)$ e $C(l_1 l_2 L; m_1 m_2)$ sono i coefficienti di WIGNER (2). l_1, l_2 e L devono soddisfare la condizione triangolare $\Delta(l_1 l_2 L)$ e la regola di parità $l_1 + l_2 + L =$ numero pari.

L'espressione analitica dei coefficienti di WIGNER (vedasi ad es. il riferimento bibliografico 4) è la seguente

$$C(l_1 l_2 L; m_1 m_2) = (-1)^{l_1 + m_1} \cdot \left[\frac{(L + l_1 - l_2)! (l_1 + l_2 - L)! (L - m)! (l_1 - m_1)! (2L + 1)}{(L - l_1 + l_2)! (l_1 + l_2 + L + 1)! (L + m)! (l_1 + m_1)! (l_2 - m_2)! (l_2 + m_2)!} \right]^{1/2} \cdot \frac{(L + l_2 + m_2)!}{(l_2 - l_2 - m_2)!} {}_3F_2(-L + l_1 - l_2, l_1 - m_1 + 1, -L - m; l_1 - l_2 - m + 1, -L - l_2 - m; 1) \quad (7)$$

dove ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$ è la serie ipergeometrica definita da:

$${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{v=0}^{\infty} \frac{a_1(v) a_2(v) a_3(v) z^v}{b_1(v) b_2(v) v!} \quad (8)$$

con

$$\begin{cases} a_n(v) = \frac{(a_n + v - 1)!}{(a_n - 1)!} \\ b_n(v) = \frac{(b_n + v - 1)!}{(b_n - 1)!} \end{cases} \quad (9)$$

Poiché, come risulta dalla (7), almeno uno degli a è un intero negativo la serie (8) si riduce sempre a un polinomio.

Nel caso particolare di $m_1 = m_2 = 0$ la (7) assume la espressione:

$$C(l_1 l_2 L; 0 0) = (-1)^{\frac{l_1 + l_2 - L}{2}} \left[\frac{2L + 1}{l_1 + l_2 + L + 1} \right]^{1/2} \cdot \frac{\tau(l_1 + l_2 + L)}{\tau(l_1 + l_2 - L) \tau(l_1 - l_2 + L) \tau(-l_1 + l_2 + L)} \quad (10)$$

dove

$$\tau(x) = \frac{\left(\frac{x}{2}\right)!}{\sqrt{x!}} \quad (11)$$

Il calcolo dei coefficienti di WIGNER è alquanto laborioso.

Alcuni autori⁽⁴⁾ hanno sviluppato le formule nei casi particolari in cui uno dei due indici l_1, l_2 valga 0, 1, 2. Noi non abbiamo trovato sviluppi per valori di l_1 o l_2 superiori a 2, necessari al fine di poter prendere in considerazione gli orbitali f . Abbiamo perciò sviluppato i relativi calcoli ed i risultati sono riportati nella seguente tabella I. Nella tabella II è riportato l'analogo sviluppo relativo alle funzioni $S_{L,M}$.

Sostituendo i risultati della tabella II nelle (5) le due distribuzioni di carica diventano:

$$\omega_a(1) = N \left[\frac{(2S_1)^{n_1} (2S_2)^{n_2}}{(2n_1)! (2n_2)!} \right]^{1/2} r_a^{n_1+n_2-2} \cdot e^{-(S_1+S_2)} r_a \sum_{L,M} \gamma_{L,M} S_{L,M}(\vartheta_{1a}, \varphi_1)$$

$$\omega_b(2) = N \left[\frac{(2S_3)^{n_3} (2S_4)^{n_4}}{(2n_3)! (2n_4)!} \right]^{1/2} r_b^{n_3+n_4-2} \cdot e^{-(S_3+S_4)} r_b \sum_{L,M} \gamma_{L,M} S_{L,M}(\vartheta_{2b}, \varphi_2) \quad (12)$$

Le (12) mostrano come ciascuna delle due distribuzioni si scinda nella somma di distribuzioni elementari caratterizzate da una parte angolare semplice del tipo $S_{L,M}(\vartheta, \varphi)$.

Poiché le funzioni $S_{L,M}$ si comportano come le rappresentazioni irriducibili del gruppo $C_{\infty, \infty}$ nell'integrale (4) rimangono soltanto quei termini che corrispondono ad uguali valori di M nelle due distribuzioni (12).

Avremo cioè:

$$I = \int \omega_a(1) \frac{1}{r_{12}} \omega_b(2) d\tau_1 d\tau_2 = \left[\frac{(2S_1)^{n_1} (2S_2)^{n_2} (2S_3)^{n_3} (2S_4)^{n_4}}{(2n_1)! (2n_2)! (2n_3)! (2n_4)!} \right]^{1/2} \cdot \sum_{L,M} N \gamma_{L,M} \sum_{L',M'} N \gamma_{L',M'} \cdot \int r_a^{n_1+n_2-2} r_b^{n_3+n_4-2} \cdot e^{-(S_1+S_2)} r_a^{-(S_1+S_2)} r_b \cdot \frac{1}{r_{12}} S_{L,M}(\vartheta_{1a}, \varphi_1) \cdot S_{L',M'}(\vartheta_{2b}, \varphi_2) d\tau_1 d\tau_2 = \left[\frac{(2S_1)^{n_1} (2S_2)^{n_2} (2S_3)^{n_3} (2S_4)^{n_4}}{(2n_1)! (2n_2)! (2n_3)! (2n_4)!} \right]^{1/2} \sum_{L',M'} N^2 \gamma_{L',M'} \cdot \int r_a^{n_1+n_2-2} r_b^{n_3+n_4-2} e^{-(S_1+S_2)} r_a^{-S_1} r_b^{-S_2} \frac{1}{r_{12}} S_{L',M'}(1) S_{L',M'}(2) d\tau = C_{n_1 n_2 n_3 n_4} \sum_{L',M'} \gamma_{L',M'} \gamma_{L',M'} J(l, l', M) \quad (13)$$

dove $C_{n_1 n_2 n_3 n_4}$ è il prodotto dei coefficienti di normalizzazione dei singoli orbitali; $\gamma_{L,M}, \gamma_{L',M'}$ sono i coefficienti che compaiono nella tabella II di decomposizione dei prodotti di orbitali, mentre $J(l, l', M)$ è un integrale elementare definito da:

$$J(l, l', M) = \frac{1}{2\pi} \int r_a^{S_1-2} r_b^{S_2-2} e^{-Z_1} r_a^{-Z_2} r_b^{-Z_3} S_{l,M}(1) S_{l',M}(2) \frac{1}{r_{12}} d\tau_1 d\tau_2 \quad (14)$$

con

$$N_1 = n_1 + n_2; N_2 = n_3 + n_4; Z_1 = S_1 + S_2; Z_2 = S_3 + S_4 \quad (15)$$

poiché $S_{l,-M}$ indica la stessa funzione di $S_{l,M}$ ruotata nello spazio, deriva che:

$$i(l, l', M) = i(l, l', -M) \quad (16)$$

Potremo perciò considerare soltanto quei valori di M tali che $M \geq 0$.

Passiamo ora a discutere il calcolo di un integrale elementare del tipo (14).

A tal fine consideriamo lo sviluppo di LEGENDRE di $\frac{1}{r_{12}}$ (v. fig. 2), che nelle $S_{l,M}$ risulta:

$$\frac{1}{r_{12}} = \sum_{L=0}^{\infty} \sum_{M=-L}^L \frac{4\pi}{2L+1} \frac{r_{<}^L}{r_{>}^{L+1}} S_{L,M}(\vartheta_{a_1}, \varphi_1) \cdot S_{L,M}(\vartheta_{a_2}, \varphi_2) \quad (17)$$

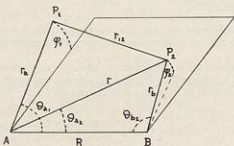


Fig. 2

Sostituendo la (17) nella (14), integrando rispetto alle variabili angolari dell'elettrone 1, e tenuto conto che le $S_{l,M}$ sono funzioni ortonormali, si ottiene:

$$J(l, l', M) = \frac{2}{2l+1} \int r_a^{2l} r_b^{2l-2} e^{-Z_1 r_a - Z_2 r_b} S_{l,M}(\vartheta_{a_1}, \varphi_2) \cdot S_{l',M}(\vartheta_{b_1}, \varphi_2) \frac{r^l}{r^{l+1}} d r_a d r_b \quad (18)$$

Operando il seguente cambiamento di variabili:

$$\begin{aligned} r_a &= E u & 0 < u < \infty \\ r_b &= E v & 0 < v < \infty \\ \cos \vartheta_{b_1} &= t & -1 < t < 1 \\ r &= E q & q = \sqrt{1 + v^2 - 2vt} \\ \cos \vartheta_{a_2} &= \xi & \xi = \frac{1 - vt}{q} \end{aligned} \quad (19)$$

sostituendo nella (18) ed integrando in φ_{2v} si ottiene:

$$\begin{aligned}
 J = (l, r, M) &= \frac{2 R^{S_1 + S_2 + 1}}{2l + 1} \int u^{S_1} v^{S_2} \dots e^{-Z_1 R u - Z_2 R v} \mathcal{P}_l^M(\xi) \mathcal{P}_r^M(\eta) \cdot \\
 &\cdot \left\{ \frac{u^l}{q^{l+1}} \right\} du dv dt = \\
 &\cdot \left\{ \frac{v^r}{u^{l+1}} \right\} \\
 &= R^{S_1 + S_2 + 1} \int u^{S_1} v^{S_2} e^{-Z_1 R u - Z_2 R v} F_{l,r,M}(u, v) du dv
 \end{aligned} \tag{20}$$

dove abbiamo posto:

$$\begin{aligned}
 F_{l,r,M}(u, v) &= \frac{2}{2l + 1} \int_{-1}^1 \mathcal{P}_l^M(\xi) \mathcal{P}_r^M(\eta) \cdot \\
 &\cdot \left\{ \frac{u^l}{q^{l+1}} \right\} dt \\
 &\cdot \left\{ \frac{v^r}{u^{l+1}} \right\}
 \end{aligned} \tag{21}$$

Nella (21) deve essere preso il termine superiore della $\left\{ \right\}$ per $u < q$; il termine inferiore per $q < u$.

Per il calcolo della (21) conviene suddividere il campo di variabilità delle u e v in quattro parti, come indicato in Fig. 3.

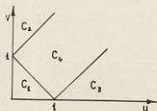


Fig. 3

In C_1 troviamo:

$$F_{l,r,M}(u, v) = \alpha(l, r, M) u^l v^r \tag{22}$$

in C_2 :

$$\begin{cases} F_{l,r,M}(u, v) = \beta(l, r, M) \frac{u^l}{q^{l+1}} & (l < r) \\ F_{l,r,M}(u, v) = 0 & (l > r) \end{cases} \tag{23}$$

in C_3 :

$$\begin{cases} F_{l,r,M}(u, v) = 0 & (l < r) \\ F_{l,r,M}(u, v) = \beta(r, l, M) \frac{v^r}{u^{l+1}} & (l \geq r) \end{cases} \tag{24}$$

L'espressione della $F(u, v)$ in C_4 è più complicata in quanto la funzione integranda cambia di forma nell'intervallo di integrazione. Si ha infatti:

$$F(u, v) = \frac{2}{2l+1} \left\{ u^l \int_{-1}^{\frac{1+v^2-u^2}{2v}} \frac{\mathcal{P}_l^M(\xi) \mathcal{P}_l^M(t)}{q^{l+1}} dt + \frac{1}{u^{l+1}} \cdot \int_{\frac{1+u^2-v^2}{2v}}^1 \mathcal{P}_l^M(\xi) \mathcal{P}_l^M(t) q^l dt \right\} \quad (25)$$

Prendendo q come variabile indipendente la (25) si scrive

$$F(u, v) = \frac{2}{(2l+1)u^{l+1}v} \left\{ u^{2l+1} \int_u^{1+v} \frac{\mathcal{P}_l^M(\xi) \mathcal{P}_l^M(t)}{q^l} dq + \int_{1-v}^u \mathcal{P}_l^M(\xi) \mathcal{P}_l^M(t) q^{l+1} dq \right\} \quad (26)$$

Nell'estremo inferiore del secondo integrale abbiamo posto $1-v$ anziché $|1-v|$ perché l'integrale indefinito è sempre un polinomio in q^2 .

Tenendo conto della relazione:

$$q(1-\xi^2)^{1/2} = v(1-t^2)^{1/2}$$

la (26) diventa

$$F(u, v) = \frac{C_{1M} \cdot C_{rM}}{(2l+1)u^{l+1}2^{2M-1}v^{M+1}} \left\{ u^{2l+1} \int_u^{1+v} \left[-(1-v^2)^2 + 2(1+v^2)q^2 - q^4 \right]^M \frac{1}{q^{L+M}} R_l^M \left(\frac{1-v^2+q^2}{2q} \right) R_l^M \left(\frac{1+v^2-q^2}{2v} \right) dq + \int_{1-v}^u \left[-(1-v^2)^2 + 2(1+v^2)q^2 - q^4 \right]^M R_l^M \left(\frac{1-v^2+q^2}{2q} \right) R_l^M \left(\frac{1+v^2-q^2}{2v} \right) q^{L-M+1} dq \right\} \quad (27)$$

dove C_{1M} , C_{rM} sono i coefficienti di normalizzazione dei polinomi associati di Legendre, mentre $R_n^M(x)$ è il polinomio di grado $x-M$ definito da:

$$R_n^M(x) = (1-x^2)^{-\frac{M}{2}} P_n^M(x)$$

Lo sviluppo della (27) conduce ad una espressione polinomiale in $u, 1+v, 1-v$, alquanto complicata.

Nel caso di $l, l' = 0, 1$ si ottiene:

$l \quad l' \quad M$

0 0 0 $F(u, v) = \frac{1}{ue} [u Q_0 + P_1]$

0 1 0 $F(u, v) = \frac{\sqrt{3}}{2 u e^2} \left\{ u [(1 + v^2) Q_0 - Q_2] + (1 + v^2) P_1 - P_2 \right\}$

1 1 0 $F(u, v) = \frac{1}{4 u^2 e^2} \left\{ u^2 [(1 - v^2) Q_2 + 2 v^2 Q_0 - Q_2] + \right.$
 $\left. + (1 - v^2) P_1 + 2 v^2 P_2 - P_3 \right\}$

1 1 1 $F(u, v) = \frac{3}{16 u^3 e^3} \left\{ u^3 [-(1 - v^2)^2 Q_{-2} + 2(1 + v^2) Q_0 - Q_2] - \right.$
 $\left. - (1 - v^2)^2 P_1 + 2(1 + v^2) P_2 - P_3 \right\}$

con

$$Q_r = \frac{(1+v)^{r+1} - u^{r+1}}{r+1} \quad P_r = \frac{u^{r+1} - (1-v)^{r+1}}{r+1}$$

Per valori maggiori di l ed l' il calcolo procede più spedatamente mediante integrazione numerica secondo la formula di integrazione di Gauss.

Posto:

$$q = \frac{1+u+v}{2} - \frac{1+v-u}{2} x$$

nel primo dei due integrali in (26), e

$$q = \frac{1+u-v}{2} - \frac{u+v-1}{2} y$$

nell'altro integrale, si ottiene:

$$F(u, v) = \frac{C_{l,M} C_{r,M}}{(2l+1) u^{l+1} e} \left\{ u^{2l+1} (1+v-u) \int_{-1}^1 P_l^M(\xi) P_l^M(\eta) \frac{d\xi}{\eta} + \right.$$

$$\left. + (u+v-1) \int_{-1}^1 P_l^M(\xi) P_l^M(\eta) \eta^{l+1} d\eta \right\} \quad (28)$$

Dette v_1, v_2, \dots, v_l le l ascisse della integrazione numerica di GAUSS e A_1, A_2, \dots, A_l i corrispondenti pesi, si ha:

$$F(u, v) = \frac{C_{l,M} C_{r,M}}{(2l+1) u^{l+1} e} \left\{ u^{2l+1} (1+v-u) \sum_i A_i P_l^M(\xi_i) P_l^M(\eta_i) \frac{1}{\eta_i} + \right.$$

$$\left. + (u+v-1) \sum_i A_i P_l^M(\xi_i) P_l^M(\eta_i) \eta_i^{l+1} \right\} \quad (29)$$

con

$$q_i = \frac{1+u+v}{2} - \frac{1+v-u}{2} v_i$$

$$q_i' = \frac{1+u-v}{2} - \frac{u+v-1}{2} v_i$$

$$t_i = \frac{1+e^2 - q_i^2}{2v}$$

$$t_i' = \frac{1+e^2 - q_i'^2}{2v}$$

$$\xi_i = \frac{1 - v t_i}{q_i}$$

$$\xi_i' = \frac{1 - v t_i'}{q_i'}$$

I valori numerici di α , β , β' che compaiono nelle (22) (23) (24) sono riportati nella seguente tabella III.

Poiché la $F(u, v)$ ha espressioni diverse nei 4 campi di variabilità delle u e v , una buona integrazione numerica dell'integrale richiederà di suddividere il campo, nelle medesime 4 parti.

Avremo allora:

$$\begin{aligned} J(l, l', M) &= R^{N_1 + N_2 + 1} \left\{ \int_{C_1} u^{N_1} v^{N_2} \dots du dv + \int_{C_2} \dots du dv + \right. \\ &\quad \left. + \int_{C_3} \dots du dv + \int_{C_4} \dots du dv = \right. \\ &= \alpha \frac{(N_1 + 1)! (N_2 + l')!}{Z_1^{N_1 + l + 1} Z_2^{N_2 + l' + 1}} \frac{1}{R^{l + l' + 1}} + R^{N_1 + N_2 + 1} \left\{ \int_{C_1} u^{N_1} v^{N_2} \cdot \right. \\ &\quad \left. e^{-Z_1 R u - Z_2 R v} \left(\beta \frac{u^l}{l!} - \alpha u^l v^l \right) du dv + \right. \\ &\quad \left. + \int_{C_2} u^{N_1} v^{N_2} e^{-Z_1 R u - Z_2 R v} \left(\beta' \frac{v^{l'}}{l'!} - \alpha u^l v^l \right) du dv + \right. \\ &\quad \left. \int_{C_3} u^{N_1} v^{N_2} e^{-Z_1 R u - Z_2 R v} (F(uv) - \alpha u^l v^l) du dv \right\} \end{aligned} \quad (30)$$

Discutiamo ora il calcolo dei tre integrali in parentesi.

In C_2 operiamo il cambiamento di variabili:

$$\begin{cases} u = \frac{X}{R(Z_1 + Z_2)} & 0 < X < \infty \\ v = 1 + u + \frac{Y}{R Z_2} & 0 < Y < \infty \end{cases} \quad |J| = \frac{1}{R^2 (Z_1 + Z_2) Z_2} \quad (31)$$

si ha allora:

$$\int_{C_1} \dots d u d v = \frac{e^{-Z_2 R}}{R^2 Z_1 (Z_1 + Z_2)} \int_0^\infty e^{-X} d X \int_0^\infty e^{-Y} u^{N_1 + l} v^{N_2 - l - 1} (\beta - \alpha v^{2l+1}) d Y$$

In questo modo possiamo applicare la formula di GAUSS-LAGUERRE.

Dette $a_1, a_2; \dots, a_N$ le ascisse di GAUSS-LAGUERRE per una integrazione ad N punti ed H_1, H_2, \dots, H_N i corrispondenti pesi, si ha:

$$\int_{C_1} \dots = \frac{e^{-Z_2 R}}{R^2 Z_1 (Z_1 + Z_2)} \sum_i H_i u_i^{N_1 + l} \sum_j H_j v_j^{N_2 - l - 1} (\beta - \alpha v_j^{2l+1}) \quad (32)$$

dove è:

$$\left\{ \begin{aligned} u_i &= \frac{a_i}{R(Z_1 + Z_2)} \\ v_{ij} &= 1 + u_i + \frac{a_j}{R Z_2} \end{aligned} \right. \quad (33)$$

Se N_1 ed N_2 fossero interi il numero dei punti da prendere per un risultato esatto dovrebbe essere $\frac{N_1 + l}{2} + 1$, e $\frac{N_2 - l}{2} + 1$ rispettivamente; nel caso di esponenti non interi il risultato sarà naturalmente tanto più preciso quanto più è elevato il numero dei punti.

Abbiamo verificato che per n ed l di valore medio, 8-10 punti conducono ad un risultato più che soddisfacente.

In C_2 , analogamente a quanto fatto in C_1 , poniamo:

$$\left\{ \begin{aligned} v &= \frac{X}{R(Z_1 + Z_2)} & 0 < X < \infty \\ u &= 1 + v + \frac{Y}{R Z_1} & 0 < Y < \infty \end{aligned} \right. \quad (34)$$

otteniamo quindi:

$$\begin{aligned} \int_{C_2} \dots &= \frac{e^{-Z_2 R}}{R^2 \cdot Z_1 (Z_1 + Z_2)} \int_0^\infty e^{-X} d X \int_0^\infty e^{-Y} u^{N_1 - l - 1} v^{N_2 + l} (\beta' - \alpha u^{2l+1}) d Y \\ &= \frac{e^{-Z_2 R}}{R^2 Z_1 (Z_1 + Z_2)} \sum_i H_i v_i^{N_2 + l} \sum_j H_j u_j^{N_1 - l - 1} (\beta' - \alpha u_j^{2l+1}) \end{aligned} \quad (35)$$

con:

$$\left\{ \begin{aligned} v_i &= \frac{a_i}{R(Z_1 + Z_2)} \\ u_{ij} &= 1 + v_i + \frac{a_j}{R Z_1} \end{aligned} \right. \quad (36)$$

In C_1 operiamo il cambiamento di variabili:

$$\begin{aligned} u &= \frac{1}{2} + \frac{X}{R(Z_1 + Z_2)} - \frac{Y}{2} & -1 < Y < 1 \\ v &= \frac{1}{2} + \frac{X}{R(Z_1 + Z_2)} + \frac{Y}{2} & 0 < X < \infty \end{aligned} \quad (37)$$

$$|J| = \frac{1}{R(Z_1 + Z_2)}$$

abbiamo allora:

$$\int_{C_1} \dots = \frac{e^{-\frac{Z_1 + Z_2}{2} R}}{R(Z_1 + Z_2)} \int_{-1}^1 e^{\frac{Z_1 - Z_2}{2} R Y} \int_0^{\infty} e^{-X u^{N_1}} v^{N_2} (F(u, v) - \alpha u^l v^l) dX \quad (38)$$

Qui integriamo la parte in X con la formula di GAUSS-LAGUERRE e la parte in Y con quella di GAUSS. Per quest'ultima siano v_1, v_2, \dots, v_l ed A_1, A_2, \dots, A_l rispettivamente, gli l punti ed i relativi pesi; si ha allora:

$$\int_{C_1} \dots = \frac{e^{-\frac{Z_1 + Z_2}{2} R}}{R(Z_1 + Z_2)} \sum_i A_i e^{\frac{Z_1 - Z_2}{2} R v_i} \sum_j H_j u_j^{N_1} v_j^{N_2} \cdot (F(u_j v_j) - \alpha u_j^l v_j^l) \quad (39)$$

con

$$\begin{aligned} u_j &= \frac{1}{2} + \frac{a_j}{R(Z_1 + Z_2)} - \frac{v_j}{2} \\ v_j &= \frac{1}{2} + \frac{a_j}{R(Z_1 + Z_2)} + \frac{v_j}{2} \end{aligned} \quad (40)$$

il calcolo di un integrale elementare risulta quindi dall'espressione

$$\begin{aligned} J(l, l', M) &= a(l, l', M) \frac{\Gamma(N_1 + l + 1) \Gamma(N_2 + l' + 1)}{Z_1^{N_1 + l + 1} Z_2^{N_2 + l' + 1}} \cdot \\ &\cdot \frac{1}{R^{l+l'+1} + R^{N_1 + N_2}} \left\{ \frac{e^{-Z_2 R}}{R Z_2 (Z_1 + Z_2)} \sum_i H_i u_i^{N_1 + l} \cdot \right. \\ &\quad \cdot \sum_j H_j v_j^{N_2 - l' - 1} (\beta - \alpha v_j^{2l' + 1}) + \\ &+ \frac{e^{-Z_1 R}}{R Z_1 (Z_1 + Z_2)} \sum_i H_i v_i^{N_1 + l} \sum_j H_j u_j^{N_2 - l' - 1} (\beta' - \alpha u_j^{2l' + 1}) + \\ &\quad \left. + \frac{e^{-\frac{Z_1 + Z_2}{2} R}}{Z_1 + Z_2} \sum_i A_i e^{\frac{Z_1 - Z_2}{2} R v_i} \sum_j H_j u_j^{N_1} v_j^{N_2} \cdot \right. \\ &\quad \left. \cdot (F(u_j v_j) - \alpha u_j^l v_j^l) \right\}. \end{aligned} \quad (41)$$

Dove gli u_j ed v_j che compaiono nelle tre sommatricie doppie sono dati rispettivamente dalle (33), (36), (40). La (41) fornisce il valore di un integrale ele-

mentare $f(L, F, M)$; un integrale coulombiano si scinde però, come già notato, in una combinazione di alcuni integrali elementari.

I coefficienti di queste combinazioni si ottengono moltiplicando fra loro quelli che compaiono nella tabella II e che corrispondono ad uguali valori di m .

Per comodità abbiamo già eseguito lo sviluppo in integrali elementari per tutti gli integrali coulombiani con orbitali s, p, d ed f , riportando i valori della tabella IV.

Ringraziamo vivamente il Prof. G.B. Bonino per lo stimolo e l'interesse mostrato a questa ricerca.

Centro Studi di Chimica applicata del Consiglio Nazionale delle Ricerche diretto dal Prof. G.B. BONINO.

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TABELLE

TABELLA I.

Decomposizione dei prodotti di funzioni armoniche sferiche.

$$N = 1/\sqrt{2\pi}.$$

$$y_0^0 y_1^0 = N \frac{\sqrt{2}}{2} y_1^0.$$

$$y_1^0 y_1^0 = N \left[\frac{\sqrt{10}}{5} y_2^0 + \frac{\sqrt{2}}{2} y_0^0 \right].$$

$$y_1^0 y_1^{\pm 1} = N \frac{\sqrt{30}}{10} y_2^{\pm 1}.$$

$$y_1^0 y_2^0 = N \left[\frac{3\sqrt{210}}{70} y_4^0 + \frac{\sqrt{10}}{5} y_0^0 \right].$$

$$y_1^0 y_2^{\pm 1} = N \left[\frac{2\sqrt{105}}{35} y_4^{\pm 1} + \frac{\sqrt{30}}{10} y_1^{\pm 1} \right].$$

$$y_1^0 y_2^{\pm 2} = N \frac{\sqrt{42}}{14} y_4^{\pm 2}.$$

$$y_1^0 y_2^0 = N \left[\frac{2\sqrt{42}}{21} y_4^0 + \frac{3\sqrt{210}}{70} y_0^0 \right].$$

$$y_1^0 y_2^{\pm 1} = N \left[\frac{\sqrt{70}}{14} y_4^{\pm 1} + \frac{2\sqrt{105}}{35} y_1^{\pm 1} \right].$$

$$y_1^0 y_2^{\pm 2} = N \frac{\sqrt{6}}{6} y_4^{\pm 2}.$$

$$\left\{ \begin{array}{l} y_1^1 y_1^1 = N \frac{\sqrt{15}}{5} y_2^1. \\ y_1^1 y_1^{-1} = N \left[-\frac{\sqrt{10}}{10} y_2^1 + \frac{\sqrt{2}}{2} y_0^0 \right]. \\ y_1^{-1} y_1^{-1} = N \frac{\sqrt{15}}{5} y_2^{-1}. \end{array} \right.$$

$$y_1^{\pm 1} y_2^0 = N \left[\frac{3\sqrt{35}}{35} y_4^{\pm 1} - \frac{\sqrt{10}}{10} y_1^{\pm 1} \right].$$

$$\left\{ \begin{array}{l} y_1^1 y_4^1 = N \frac{\sqrt{21}}{7} y_3^2, \\ y_1^1 y_4^{-1} = y_1^{-1} y_2^1 = N \left[-\frac{3\sqrt{70}}{70} y_3^2 + \frac{\sqrt{30}}{10} y_1^2 \right], \\ y_1^{-1} y_2^{-1} = N \frac{\sqrt{21}}{7} y_3^{-1}. \end{array} \right.$$

$$\left\{ \begin{array}{l} y_1^1 y_2^2 = N \frac{3\sqrt{14}}{14} y_3^2, \\ y_1^1 y_3^{-1} = N \left[-\frac{\sqrt{210}}{70} y_3^{-1} + \frac{\sqrt{15}}{5} y_1^{-1} \right], \\ y_1^{-1} y_2^2 = N \left[-\frac{\sqrt{210}}{70} y_3^2 + \frac{\sqrt{15}}{5} y_1^1 \right], \\ y_1^{-1} y_3^{-1} = N \frac{3\sqrt{14}}{14} y_3^{-2}. \end{array} \right.$$

$$y_1^{21} y_3^2 = N \left[\frac{105}{21} y_4^{21} - \frac{3\sqrt{70}}{70} y_2^{21} \right].$$

$$\left\{ \begin{array}{l} y_1^1 y_4^1 = N \left[\frac{\sqrt{70}}{14} y_4^1 - \frac{\sqrt{210}}{70} y_2^1 \right], \\ y_1^1 y_4^{-1} = y_1^{-1} y_4^1 = N \left[-\frac{\sqrt{7}}{7} y_4^1 + \frac{3\sqrt{35}}{35} y_2^1 \right], \\ y_1^{-1} y_3^{-1} = N \left[\frac{\sqrt{70}}{14} y_4^{-1} - \frac{\sqrt{210}}{70} y_2^{-1} \right]. \end{array} \right.$$

$$\left\{ \begin{array}{l} y_1^1 y_3^2 = N \frac{\sqrt{2}}{2} y_4^2, \\ y_1^1 y_3^{-1} = N \left[-\frac{\sqrt{14}}{14} y_4^{-1} + \frac{\sqrt{21}}{7} y_2^{-1} \right], \\ y_1^{-1} y_3^2 = N \left[-\frac{\sqrt{14}}{14} y_4^2 + \frac{\sqrt{21}}{7} y_2^1 \right], \\ y_1^{-1} y_3^{-1} = N \frac{\sqrt{2}}{2} y_4^{-1}. \end{array} \right.$$

$$\left\{ \begin{array}{l}
 y_1^1 y_2^1 = N \frac{\sqrt{6}}{3} y_4^1, \\
 y_1^1 y_2^{-1} = N \left[-\frac{\sqrt{42}}{42} y_4^{-1} + \frac{3\sqrt{14}}{14} y_2^{-1} \right], \\
 y_1^{-1} y_2^1 = N \left[-\frac{\sqrt{42}}{42} y_4^1 + \frac{3\sqrt{14}}{14} y_2^1 \right], \\
 y_1^{-1} y_2^{-1} = N \frac{\sqrt{6}}{3} y_4^{-1}.
 \end{array} \right.$$

$$y_2^0 y_3^0 = N \left[\frac{3\sqrt{2}}{7} y_4^0 + \frac{\sqrt{10}}{7} y_5^0 + \frac{\sqrt{2}}{2} y_6^0 \right],$$

$$y_2^0 y_3^{+1} = N \left[\frac{\sqrt{15}}{7} y_4^{+1} + \frac{\sqrt{10}}{14} y_2^{+1} \right],$$

$$y_2^0 y_3^{+2} = N \left[\frac{\sqrt{30}}{14} y_4^{+2} - \frac{\sqrt{10}}{7} y_2^{+2} \right],$$

$$y_2^0 y_3^0 = N \left[\frac{5\sqrt{770}}{231} y_4^0 + \frac{2\sqrt{10}}{15} y_5^0 + \frac{3\sqrt{210}}{70} y_6^0 \right],$$

$$y_2^0 y_3^{+1} = N \left[\frac{5\sqrt{77}}{77} y_4^{+1} + \frac{\sqrt{10}}{10} y_5^{+1} + \frac{3\sqrt{35}}{35} y_6^{+1} \right],$$

$$y_2^0 y_3^{+2} = N \frac{\sqrt{110}}{22} y_4^{+2},$$

$$y_2^0 y_3^{+3} = N \left[\frac{\sqrt{110}}{33} y_4^{+3} - \frac{\sqrt{10}}{6} y_2^{+3} \right],$$

$$\left\{ \begin{array}{l}
 y_2^1 y_3^1 = N \left[\frac{2\sqrt{5}}{7} y_4^1 + \frac{\sqrt{15}}{7} y_2^1 \right], \\
 y_2^1 y_3^{-1} = N \left[-\frac{2\sqrt{2}}{7} y_4^0 + \frac{\sqrt{10}}{14} y_5^0 + \frac{\sqrt{2}}{2} y_6^0 \right], \\
 y_2^{-1} y_3^{-1} = N \left[\frac{2\sqrt{5}}{7} y_4^{-1} + \frac{\sqrt{15}}{7} y_2^{-1} \right].
 \end{array} \right.$$

$$\left\{ \begin{aligned}
 y_4^1 y_3^2 &= N \frac{\sqrt{70}}{14} y_4^1, \\
 y_2^1 y_3^{-2} &= N \left[-\frac{\sqrt{10}}{14} y_4^{-1} + \frac{\sqrt{15}}{7} y_2^{-1} \right], \\
 y_4^{-1} y_3^2 &= N \left[-\frac{\sqrt{10}}{14} y_4^1 + \frac{\sqrt{15}}{7} y_2^1 \right], \\
 y_2^{-1} y_3^{-2} &= N \frac{\sqrt{70}}{14} y_4^{-1}, \\
 y_2^1 y_3^0 &= N \left[\frac{10\sqrt{154}}{231} y_4^1 + \frac{\sqrt{5}}{15} y_3^1 - \frac{3\sqrt{70}}{70} y_4^1 \right], \\
 y_2^1 y_3^1 &= N \left[\frac{5\sqrt{66}}{66} y_4^1 + \frac{\sqrt{6}}{6} y_3^1 \right], \\
 y_2^1 y_3^{-1} - y_3^{-1} y_4^1 &= N \left[-\frac{5\sqrt{385}}{231} y_4^1 + \frac{\sqrt{5}}{15} y_3^1 + \frac{2\sqrt{105}}{35} y_3^1 \right], \\
 y_4^{-1} y_3^{-1} &= N \left[\frac{5\sqrt{66}}{66} y_4^{-1} + \frac{\sqrt{6}}{6} y_3^{-1} \right], \\
 y_2^1 y_3^2 &= N \left[\frac{2\sqrt{110}}{33} y_4^1 + \frac{\sqrt{10}}{6} y_3^1 \right], \\
 y_2^1 y_3^{-2} &= N \left[-\frac{2\sqrt{1155}}{231} y_4^{-1} + \frac{\sqrt{6}}{6} y_3^{-1} + \frac{\sqrt{21}}{7} y_1^{-1} \right], \\
 y_2^{-1} y_3^2 &= N \left[-\frac{2\sqrt{1155}}{231} y_4^1 + \frac{\sqrt{6}}{6} y_3^1 + \frac{\sqrt{21}}{7} y_1^1 \right], \\
 y_2^{-1} y_3^{-2} &= N \left[\frac{2\sqrt{110}}{33} y_4^{-1} + \frac{\sqrt{10}}{6} y_3^{-1} \right], \\
 y_2^1 y_3^0 &= N \frac{\sqrt{330}}{33} y_4^1, \\
 y_2^1 y_3^{-2} &= N \left[-\frac{\sqrt{110}}{66} y_4^{-1} + \frac{\sqrt{10}}{6} y_3^{-1} \right], \\
 y_2^{-1} y_3^2 &= N \left[-\frac{\sqrt{110}}{66} y_4^1 + \frac{\sqrt{10}}{6} y_3^1 \right], \\
 y_2^{-1} y_3^{-2} &= N \frac{\sqrt{330}}{33} y_4^{-1}.
 \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_1^2 y_2^2 &= N \frac{\sqrt{35}}{7} y_4^2, \\ y_2^2 y_3^2 &= N \left[\frac{\sqrt{2}}{14} y_4^2 - \frac{\sqrt{10}}{7} y_2^2 + \frac{\sqrt{2}}{2} y_6^2 \right], \\ y_4^2 y_5^2 &= N \frac{\sqrt{35}}{7} y_4^2. \end{aligned} \right.$$

$$y_2^2 y_3^2 = N \left[\frac{5\sqrt{22}}{66} y_5^2 - \frac{\sqrt{2}}{3} y_6^2 \right].$$

$$\left\{ \begin{aligned} y_1^2 y_3^2 &= N \left[\frac{5\sqrt{11}}{33} y_4^2 - \frac{1}{3} y_6^2 \right], \\ y_1^2 y_5^2 &= N \left[-\frac{5\sqrt{462}}{462} y_4^2 + \frac{2\sqrt{15}}{15} y_1^2 - \frac{\sqrt{210}}{70} y_1^2 \right], \\ y_2^2 y_3^2 &= N \left[-\frac{5\sqrt{462}}{462} y_4^2 + \frac{2\sqrt{15}}{15} y_2^2 - \frac{\sqrt{210}}{70} y_1^2 \right], \\ y_4^2 y_5^2 &= N \left[\frac{5\sqrt{11}}{33} y_4^2 - \frac{1}{3} y_6^2 \right]. \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_1^2 y_2^2 &= N \frac{\sqrt{55}}{11} y_4^2, \\ y_2^2 y_3^2 - y_4^2 y_5^2 &= N \left[\frac{5\sqrt{154}}{462} y_4^2 - \frac{\sqrt{2}}{3} y_6^2 + \frac{\sqrt{42}}{14} y_1^2 \right], \\ y_2^2 y_5^2 &= N \frac{\sqrt{55}}{11} y_4^2. \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_1^2 y_4^2 &= N \frac{5\sqrt{39}}{39} y_5^2, \\ y_2^2 y_3^2 &= N \left[\frac{\sqrt{770}}{462} y_5^2 - \frac{1}{3} y_1^2 + \frac{3\sqrt{350}}{70} y_1^2 \right], \\ y_4^2 y_5^2 &= N \left[\frac{\sqrt{770}}{462} y_5^2 - \frac{1}{3} y_1^2 + \frac{3\sqrt{350}}{70} y_1^2 \right], \\ y_2^2 y_5^2 &= N \frac{5\sqrt{39}}{39} y_5^2. \end{aligned} \right.$$

$$y_4^2 y_5^2 = N \left[\frac{50\sqrt{26}}{429} y_6^2 + \frac{3\sqrt{2}}{11} y_4^2 + \frac{2\sqrt{10}}{15} y_2^2 + \frac{\sqrt{2}}{2} y_6^2 \right].$$

$$\begin{aligned}
 y_3^0 y_3^{+1} &= N \left[\frac{25\sqrt{91}}{429} y_4^{+1} + \frac{\sqrt{30}}{22} y_4^{+1} + \frac{\sqrt{5}}{15} y_4^{+1} \right], \\
 y_3^0 y_3^{+2} &= N \left[\frac{20\sqrt{91}}{429} y_4^{+2} - \frac{\sqrt{6}}{22} y_4^{+2} - \frac{\sqrt{2}}{3} y_4^{+2} \right], \\
 y_3^0 y_3^{+3} &= N \left[\frac{5\sqrt{546}}{429} y_4^{+3} - \frac{3\sqrt{14}}{22} y_4^{+3} \right], \\
 \left\{ \begin{aligned}
 y_3^1 y_3^1 &= N \left[\frac{5\sqrt{2730}}{429} y_4^2 + \frac{2\sqrt{5}}{11} y_4^2 + \frac{2\sqrt{15}}{15} y_4^2 \right], \\
 y_3^1 y_3^1 - y_3^1 y_3^{-1} &= N \left[-\frac{25\sqrt{26}}{286} y_4^2 \right], \\
 y_3^{-1} y_3^{-1} &= N \left[\frac{5\sqrt{2730}}{429} y_4^{-2} + \frac{2\sqrt{5}}{11} y_4^{-2} + \frac{2\sqrt{15}}{15} y_4^{-2} \right].
 \end{aligned} \right. \\
 \left\{ \begin{aligned}
 y_3^1 y_3^2 &= N \left[\frac{5\sqrt{273}}{143} y_4^2 + \frac{\sqrt{7}}{11} y_4^2 \right], \\
 y_3^2 y_3^{-2} &= N \left[-\frac{5\sqrt{2730}}{858} y_4^{-1} + \frac{4}{11} y_4^{-1} + \frac{\sqrt{6}}{6} y_4^{-1} \right], \\
 y_3^{-1} y_3^2 &= N \left[-\frac{5\sqrt{2730}}{858} y_4^1 + \frac{4}{11} y_4^1 + \frac{\sqrt{6}}{6} y_4^1 \right], \\
 y_3^{-1} y_3^{-2} &= N \left[\frac{5\sqrt{273}}{143} y_4^{-3} + \frac{\sqrt{7}}{11} y_4^{-3} \right].
 \end{aligned} \right. \\
 \left\{ \begin{aligned}
 y_3^1 y_3^3 &= N \left[\frac{5\sqrt{1365}}{429} y_4^3 - \frac{\sqrt{21}}{11} y_4^3 \right], \\
 y_3^1 y_3^{-3} &= N \left[-\frac{5\sqrt{182}}{429} y_4^{-3} + \frac{3\sqrt{3}}{11} y_4^{-3} - \frac{1}{3} y_4^{-3} \right], \\
 y_3^{-1} y_3^3 &= N \left[-\frac{5\sqrt{182}}{429} y_4^3 + \frac{3\sqrt{3}}{11} y_4^3 - \frac{1}{3} y_4^3 \right], \\
 y_3^{-3} y_3^{-3} &= N \left[\frac{5\sqrt{1365}}{429} y_4^{-4} - \frac{\sqrt{21}}{11} y_4^{-4} \right].
 \end{aligned} \right. \\
 \left\{ \begin{aligned}
 y_3^2 y_3^2 &= N \left[\frac{10\sqrt{91}}{143} y_4^4 + \frac{\sqrt{35}}{11} y_4^4 \right], \\
 y_3^2 y_3^{-2} = y_3^{-2} y_3^2 &= N \left[\frac{5\sqrt{26}}{143} y_4^0 - \frac{7\sqrt{2}}{22} y_4^0 + \frac{\sqrt{2}}{2} y_4^0 \right], \\
 y_3^{-2} y_3^{-2} &= N \left[\frac{10\sqrt{91}}{143} y_4^{-4} + \frac{\sqrt{35}}{11} y_4^{-4} \right].
 \end{aligned} \right.
 \end{aligned}$$

$$\left\{ \begin{aligned} y_1^2 y_2^2 &= N \frac{5\sqrt{3003}}{429} y_6^2, \\ y_1^3 y_2^{-3} &= N \left[\frac{5\sqrt{182}}{858} y_6^{-1} - \frac{3\sqrt{15}}{33} y_4^{-1} + \frac{\sqrt{10}}{6} y_2^{-1} \right], \\ y_1^{-2} y_2^2 &= N \left[\frac{5\sqrt{182}}{858} y_6^1 - \frac{3\sqrt{15}}{33} y_4^1 + \frac{\sqrt{10}}{6} y_2^{-1} \right], \\ y_1^{-2} y_2^{-2} &= N \frac{5\sqrt{3003}}{429} y_6^{-2}. \end{aligned} \right.$$

$$\left\{ \begin{aligned} y_1^3 y_2^2 &= N \frac{5\sqrt{6006}}{429} y_6^4, \\ y_1^{-2} y_2^2 - y_1^2 y_2^{-2} &= N \left[-\frac{5\sqrt{26}}{858} y_6^2 + \frac{3\sqrt{2}}{22} y_4^2 - \frac{\sqrt{10}}{6} y_2^2 + \frac{\sqrt{2}}{2} y_6^2 \right], \\ y_1^{-2} y_2^{-2} &= N \frac{5\sqrt{6006}}{429} y_6^{-4}. \end{aligned} \right.$$

TABELLA II.

Decomposizioni dei prodotti di funzioni reali $S_{i,m}$.

$$N = 1/\sqrt{2\pi}.$$

$$s s = N \frac{\sqrt{2}}{2} S_{0,0}.$$

$$s P_1 = N \frac{\sqrt{2}}{2} S_{1,1}.$$

$$s d_0 = N \frac{\sqrt{2}}{2} S_{2,0}.$$

$$s d_{-1} = N \frac{\sqrt{2}}{2} S_{2,-1}.$$

$$s d_{-2} = N \frac{\sqrt{2}}{2} S_{2,-2}.$$

$$s f_1 = N \frac{\sqrt{2}}{2} S_{3,1}.$$

$$s f_2 = N \frac{\sqrt{2}}{2} S_{3,2}.$$

$$s f_3 = N \frac{\sqrt{2}}{2} S_{3,3}.$$

$$P_0 P_0 = N \left[\frac{\sqrt{10}}{5} S_{2,0} + \frac{\sqrt{2}}{2} S_{0,0} \right].$$

$$P_0 P_{-1} = N \frac{\sqrt{30}}{10} S_{1,-1}.$$

$$P_0 d_0 = N \left[\frac{3\sqrt{210}}{70} S_{3,0} + \frac{\sqrt{10}}{5} S_{1,0} \right].$$

$$P_0 d_{-1} = N \left[\frac{2\sqrt{105}}{35} S_{3,-1} + \frac{\sqrt{30}}{10} S_{1,-1} \right].$$

$$P_0 d_{-2} = N \frac{\sqrt{42}}{14} S_{1,-2}.$$

$$s P_0 = N \frac{\sqrt{2}}{2} S_{1,0}.$$

$$s P_{-1} = N \frac{\sqrt{2}}{2} S_{1,-1}.$$

$$s d_1 = N \frac{\sqrt{2}}{2} S_{2,1}.$$

$$s d_2 = N \frac{\sqrt{2}}{2} S_{2,2}.$$

$$s f_0 = N \frac{\sqrt{2}}{2} S_{3,0}.$$

$$s f_{-1} = N \frac{\sqrt{2}}{2} S_{3,-1}.$$

$$s f_{-2} = N \frac{\sqrt{2}}{2} S_{3,-2}.$$

$$s f_{-3} = N \frac{\sqrt{2}}{2} S_{3,-3}.$$

$$P_0 P_1 = N \frac{\sqrt{30}}{10} S_{2,1}.$$

$$P_0 d_1 = N \left[\frac{2\sqrt{105}}{35} S_{3,1} + \frac{\sqrt{30}}{10} S_{1,1} \right].$$

$$P_0 d_2 = N \frac{\sqrt{42}}{14} S_{2,2}.$$

$$P_0 f_0 = N \left[\frac{2\sqrt{42}}{21} S_{4,0} + \frac{3\sqrt{210}}{70} S_{2,0} \right].$$

$$p_0 l_1 = N \left[\frac{\sqrt{70}}{14} s_{0,1} + \frac{2\sqrt{105}}{35} s_{0,1} \right].$$

$$p_0 l_{-1} = N \left[\frac{\sqrt{70}}{14} s_{0,-1} + \frac{2\sqrt{105}}{35} s_{0,-1} \right].$$

$$p_0 l_2 = N \left[\frac{\sqrt{14}}{7} s_{0,2} + \frac{\sqrt{42}}{14} s_{0,2} \right].$$

$$p_0 l_{-2} = N \left[\frac{\sqrt{14}}{7} s_{0,-2} + \frac{\sqrt{42}}{14} s_{0,-2} \right].$$

$$p_0 l_3 = N \frac{\sqrt{6}}{6} s_{0,3}.$$

$$p_0 l_{-3} = N \frac{\sqrt{6}}{6} s_{0,-3}.$$

$$p_1 p_1 = N \left[\frac{\sqrt{30}}{10} s_{1,1} - \frac{\sqrt{10}}{10} s_{1,0} + \frac{\sqrt{2}}{2} s_{0,0} \right].$$

$$p_1 p_{-1} = N \frac{\sqrt{30}}{10} s_{1,-1}.$$

$$p_1 d_0 = N \left[\frac{3\sqrt{35}}{35} s_{1,1} - \frac{\sqrt{10}}{10} s_{1,1} \right].$$

$$p_1 d_1 = N \left[\frac{\sqrt{42}}{14} s_{1,2} - \frac{3\sqrt{70}}{70} s_{1,0} + \frac{\sqrt{30}}{10} s_{0,0} \right].$$

$$p_1 d_{-1} = N \frac{\sqrt{42}}{14} s_{1,-2}$$

$$p_1 d_2 = N \left[\frac{3\sqrt{7}}{14} s_{1,3} - \frac{\sqrt{105}}{70} s_{1,1} + \frac{\sqrt{30}}{10} s_{1,1} \right].$$

$$p_1 d_{-2} = N \left[\frac{3\sqrt{7}}{14} s_{1,-3} - \frac{\sqrt{105}}{70} s_{1,-1} + \frac{\sqrt{30}}{10} s_{1,-1} \right].$$

$$p_1 l_0 = N \left[\frac{\sqrt{105}}{21} s_{1,1} - \frac{3\sqrt{70}}{70} s_{2,1} \right].$$

$$p_1 l_1 = N \left[\frac{\sqrt{35}}{14} s_{1,2} - \frac{\sqrt{105}}{70} s_{2,2} - \frac{\sqrt{7}}{7} s_{0,0} + \frac{3\sqrt{35}}{35} s_{1,0} \right].$$

$$p_1 l_{-1} = N \left[\frac{\sqrt{35}}{14} s_{1,-2} - \frac{\sqrt{105}}{70} s_{2,-2} \right].$$

$$p_1 l_2 = N \left[\frac{1}{2} s_{1,3} - \frac{\sqrt{7}}{14} s_{1,1} + \frac{\sqrt{42}}{14} s_{2,1} \right].$$

$$p_1 l_{-2} = N \left[\frac{1}{2} s_{1,-3} - \frac{\sqrt{7}}{14} s_{1,-1} + \frac{\sqrt{42}}{14} s_{2,-1} \right].$$

$$p_1 l_3 = N \left[\frac{\sqrt{3}}{3} s_{1,4} - \frac{\sqrt{21}}{42} s_{1,2} + \frac{3\sqrt{7}}{14} s_{2,2} \right].$$

$$p_1 l_{-3} = N \left[\frac{\sqrt{3}}{3} s_{1,-4} - \frac{\sqrt{21}}{42} s_{1,-2} + \frac{3\sqrt{7}}{14} s_{2,-2} \right].$$

$$p_{-1} p_{-1} = -N \left[\frac{\sqrt{30}}{10} s_{2,2} + \frac{\sqrt{10}}{10} s_{2,3} - \frac{\sqrt{2}}{2} s_{2,3} \right].$$

$$p_{-1} d_0 = N \left[\frac{3\sqrt{35}}{35} s_{2,-1} - \frac{\sqrt{10}}{10} s_{1,-1} \right]. \quad p_{-1} d_1 = N \frac{\sqrt{42}}{14} s_{2,-2}.$$

$$p_{-1} d_{-1} = -N \left[\frac{\sqrt{42}}{14} s_{2,-1} + \frac{3\sqrt{70}}{70} s_{2,0} - \frac{\sqrt{30}}{10} s_{1,0} \right].$$

$$p_{-1} d_2 = N \left[\frac{3\sqrt{7}}{14} s_{2,-2} - \frac{\sqrt{105}}{70} s_{2,-1} - \frac{\sqrt{30}}{10} s_{1,-1} \right].$$

$$p_{-1} d_{-2} = -N \left[\frac{3\sqrt{7}}{14} s_{2,2} + \frac{\sqrt{105}}{70} s_{2,1} - \frac{\sqrt{30}}{10} s_{1,1} \right].$$

$$p_{-2} f_0 = N \left[\frac{\sqrt{105}}{21} s_{2,-1} - \frac{3\sqrt{70}}{70} s_{2,-1} \right].$$

$$p_{-1} f_1 = N \left[\frac{\sqrt{35}}{14} s_{2,-1} - \frac{\sqrt{105}}{70} s_{1,-1} \right]$$

$$p_{-1} f_{-1} = -N \left[\frac{\sqrt{35}}{14} s_{2,2} - \frac{\sqrt{105}}{70} s_{2,2} + \frac{\sqrt{7}}{7} s_{2,0} - \frac{3\sqrt{35}}{35} s_{2,0} \right].$$

$$p_{-1} f_2 = N \left[\frac{\sqrt{16}}{8} s_{2,-2} + \frac{\sqrt{7}}{14} s_{2,-1} - \frac{\sqrt{42}}{14} s_{1,-1} \right].$$

$$p_{-1} f_{-2} = -N \left[\frac{\sqrt{16}}{8} s_{2,2} + \frac{\sqrt{7}}{14} s_{2,1} - \frac{\sqrt{42}}{14} s_{2,1} \right].$$

$$p_{-1} f_3 = N \left[\frac{\sqrt{12}}{6} s_{2,-2} + \frac{\sqrt{21}}{42} s_{2,-1} - \frac{3\sqrt{7}}{14} s_{2,-1} \right].$$

$$p_{-1} f_{-3} = -N \left[\frac{\sqrt{12}}{6} s_{2,2} + \frac{\sqrt{21}}{42} s_{2,2} - \frac{3\sqrt{7}}{14} s_{2,2} \right].$$

$$d_0 d_0 = N \left[\frac{3\sqrt{2}}{7} s_{2,0} + \frac{\sqrt{10}}{7} s_{2,0} + \frac{\sqrt{2}}{2} s_{2,0} \right].$$

$$d_0 d_1 = N \left[\frac{\sqrt{15}}{7} s_{2,1} + \frac{\sqrt{10}}{14} s_{2,1} \right].$$

$$d_0 d_{-1} = N \left[\frac{\sqrt{15}}{7} s_{2,-1} + \frac{\sqrt{10}}{14} s_{2,-1} \right].$$

$$d_0 d_2 = N \left[\frac{\sqrt{30}}{14} s_{2,2} - \frac{\sqrt{10}}{7} s_{2,2} \right].$$

$$d_6 d_{-1} = N \left[\frac{\sqrt{30}}{14} s_{k-1} - \frac{\sqrt{10}}{7} s_{k-1} \right].$$

$$d_6 t_6 = N \left[\frac{5\sqrt{770}}{231} s_{k,6} + \frac{2\sqrt{10}}{15} s_{k,6} + \frac{3\sqrt{210}}{70} s_{k,6} \right].$$

$$d_6 t_1 = N \left[\frac{5\sqrt{77}}{77} s_{k,1} + \frac{\sqrt{10}}{10} s_{k,1} + \frac{3\sqrt{35}}{35} s_{k,1} \right].$$

$$d_6 L_{-1} = N \left[\frac{5\sqrt{77}}{77} s_{k-1} + \frac{\sqrt{10}}{16} s_{k-1} + \frac{3\sqrt{35}}{35} s_{k-1} \right].$$

$$d_6 t_2 = N \frac{\sqrt{110}}{22} s_{k,2}.$$

$$d_6 L_{-2} = N \frac{\sqrt{110}}{22} s_{k-2}.$$

$$d_6 t_3 = N \left[\frac{\sqrt{110}}{33} s_{k,3} - \frac{\sqrt{10}}{6} s_{k,3} \right].$$

$$d_6 L_{-3} = N \left[\frac{\sqrt{110}}{33} s_{k-3} - \frac{\sqrt{10}}{6} s_{k-3} \right].$$

$$d_1 d_1 = N \left[\frac{\sqrt{10}}{7} s_{k,1} + \frac{\sqrt{30}}{14} s_{k,1} - \frac{2\sqrt{2}}{7} s_{k,6} + \frac{\sqrt{10}}{14} s_{k,6} + \frac{\sqrt{2}}{2} s_{k,6} \right].$$

$$d_1 d_{-1} = N \left[\frac{\sqrt{10}}{7} s_{k-1} + \frac{\sqrt{30}}{14} s_{k-1} \right].$$

$$d_1 d_2 = N \left[\frac{\sqrt{35}}{14} s_{k,2} - \frac{\sqrt{5}}{14} s_{k,1} + \frac{\sqrt{30}}{14} s_{k,1} \right].$$

$$d_1 d_{-2} = N \left[\frac{\sqrt{35}}{14} s_{k-2} - \frac{\sqrt{5}}{14} s_{k-1} + \frac{\sqrt{30}}{14} s_{k-1} \right].$$

$$d_1 t_6 = N \left[\frac{10\sqrt{154}}{231} s_{k,1} + \frac{\sqrt{5}}{15} s_{k,1} - \frac{3\sqrt{70}}{70} s_{k,1} \right].$$

$$d_1 t_1 = N \left[\frac{5\sqrt{33}}{66} s_{k,1} + \frac{\sqrt{3}}{6} s_{k,2} - \frac{5\sqrt{385}}{231} s_{k,6} + \frac{\sqrt{5}}{15} s_{k,6} + \frac{2\sqrt{105}}{35} s_{k,6} \right].$$

$$d_1 L_{-1} = N \left[\frac{5\sqrt{33}}{66} s_{k-2} + \frac{\sqrt{3}}{6} s_{k-2} \right].$$

$$d_1 t_2 = N \left[\frac{2\sqrt{55}}{33} s_{k,2} + \frac{\sqrt{5}}{6} s_{k,2} - \frac{\sqrt{2310}}{231} s_{k,1} + \frac{\sqrt{3}}{6} s_{k,1} + \frac{\sqrt{42}}{14} s_{k,1} \right].$$

$$d_1 L_{-2} = N \left[\frac{2\sqrt{55}}{33} s_{k-1} + \frac{\sqrt{5}}{6} s_{k-2} - \frac{\sqrt{2310}}{231} s_{k-1} + \frac{\sqrt{3}}{6} s_{k-1} + \frac{\sqrt{42}}{14} s_{k-1} \right].$$

$$d_1 f_5 = N \left[\frac{\sqrt{165}}{33} s_{3,4} - \frac{\sqrt{55}}{66} s_{3,2} + \frac{\sqrt{5}}{6} s_{3,1} \right].$$

$$d_1 f_3 = N \left[\frac{\sqrt{165}}{33} s_{2,4} - \frac{\sqrt{55}}{66} s_{2,2} \right].$$

$$d_{-1} d_{-1} = -N \left[\frac{\sqrt{10}}{7} s_{4,4} + \frac{\sqrt{30}}{14} s_{3,3} + \frac{2\sqrt{2}}{7} s_{3,0} - \frac{\sqrt{10}}{4} s_{3,0} - \frac{\sqrt{2}}{2} s_{0,0} \right].$$

$$d_{-1} d_2 = N \left[\frac{\sqrt{35}}{14} s_{4,3} + \frac{\sqrt{5}}{14} s_{4,1} - \frac{\sqrt{30}}{14} s_{2,1} \right].$$

$$d_{-1} d_{-2} = -N \left[\frac{\sqrt{35}}{14} s_{3,3} + \frac{\sqrt{5}}{14} s_{3,1} - \frac{\sqrt{30}}{14} s_{1,1} \right].$$

$$d_{-1} f_6 = N \left[\frac{10\sqrt{154}}{231} s_{3,1} + \frac{\sqrt{5}}{15} s_{3,1} - \frac{3\sqrt{70}}{70} s_{1,1} \right];$$

$$d_{-1} f_1 = N \left[\frac{5\sqrt{33}}{66} s_{2,4} + \frac{\sqrt{3}}{6} s_{2,2} \right].$$

$$d_{-1} f_{-1} = -N \left[\frac{5\sqrt{33}}{66} s_{2,2} + \frac{\sqrt{3}}{6} s_{2,2} + \frac{5\sqrt{385}}{231} s_{2,0} - \frac{\sqrt{5}}{15} s_{2,0} - \frac{2\sqrt{105}}{35} s_{1,0} \right].$$

$$d_{-1} f_2 = N \left[\frac{2\sqrt{55}}{33} s_{2,3} + \frac{\sqrt{5}}{6} s_{2,3} - \frac{\sqrt{2310}}{231} s_{2,1} + \frac{\sqrt{3}}{6} s_{2,1} + \frac{\sqrt{42}}{14} s_{1,1} \right].$$

$$d_{-1} f_{-2} = -N \left[\frac{2\sqrt{55}}{33} s_{2,3} + \frac{\sqrt{5}}{6} s_{2,3} + \frac{\sqrt{2310}}{231} s_{2,1} - \frac{\sqrt{3}}{6} s_{2,1} - \frac{\sqrt{42}}{14} s_{1,1} \right].$$

$$d_{-1} f_3 = N \left[\frac{\sqrt{165}}{33} s_{2,4} - \frac{\sqrt{55}}{66} s_{2,4} + \frac{\sqrt{5}}{6} s_{2,2} \right].$$

$$d_{-1} f_{-3} = -N \left[\frac{\sqrt{165}}{33} s_{3,4} + \frac{\sqrt{35}}{66} s_{3,2} - \frac{\sqrt{5}}{6} s_{3,2} \right].$$

$$d_4 d_2 = N \left[\frac{\sqrt{70}}{14} s_{4,4} + \frac{\sqrt{2}}{14} s_{4,0} - \frac{\sqrt{10}}{7} s_{2,0} + \frac{\sqrt{2}}{2} s_{0,0} \right].$$

$$d_4 d_{-2} = N \frac{\sqrt{70}}{14} s_{4,4} \qquad d_4 f_6 = N \left[\frac{5\sqrt{22}}{66} s_{3,2} - \frac{\sqrt{2}}{3} s_{3,2} \right].$$

$$d_4 f_1 = N \left[\frac{5\sqrt{22}}{66} s_{3,2} - \frac{\sqrt{2}}{6} s_{3,2} - \frac{5\sqrt{231}}{462} s_{3,1} + \frac{\sqrt{30}}{15} s_{3,1} - \frac{\sqrt{105}}{70} s_{1,1} \right].$$

$$d_4 f_{-1} = N \left[\frac{5\sqrt{22}}{66} s_{3,2} - \frac{\sqrt{2}}{6} s_{3,2} - \frac{5\sqrt{231}}{462} s_{3,1} + \frac{\sqrt{30}}{15} s_{3,1} - \frac{\sqrt{105}}{70} s_{1,1} \right].$$

$$d_4 t_4 = N \left[\frac{\sqrt{165}}{33} S_{3,4} + \frac{5\sqrt{154}}{462} S_{3,3} - \frac{\sqrt{2}}{3} S_{3,2} + \frac{\sqrt{42}}{14} S_{3,1} \right].$$

$$d_4 t_{-4} = N \frac{\sqrt{165}}{33} S_{3,-4}.$$

$$d_4 t_3 = N \left[\frac{5\sqrt{78}}{78} S_{3,3} + \frac{\sqrt{385}}{462} S_{3,2} - \frac{\sqrt{2}}{6} S_{3,1} + \frac{3\sqrt{7}}{14} S_{3,0} \right].$$

$$d_4 t_{-3} = N \left[\frac{5\sqrt{78}}{78} S_{3,-3} + \frac{\sqrt{385}}{462} S_{3,-2} - \frac{\sqrt{2}}{6} S_{3,-1} + \frac{3\sqrt{7}}{14} S_{3,-0} \right].$$

$$d_{-4} d_{-4} = -N \left[\frac{\sqrt{70}}{14} S_{4,4} - \frac{\sqrt{2}}{14} S_{4,3} + \frac{\sqrt{10}}{7} S_{4,2} - \frac{\sqrt{2}}{2} S_{4,1} \right].$$

$$d_{-4} t_6 = N \left[\frac{5\sqrt{22}}{66} S_{3,-1} - \frac{\sqrt{2}}{3} S_{3,-2} \right].$$

$$d_{-4} t_4 = N \left[\frac{5\sqrt{22}}{66} S_{3,-3} - \frac{\sqrt{2}}{6} S_{3,-1} + \frac{5\sqrt{231}}{462} S_{3,-1} - \frac{\sqrt{30}}{15} S_{3,-1} + \frac{\sqrt{105}}{70} S_{3,-1} \right].$$

$$d_{-4} t_{-4} = -N \left[\frac{5\sqrt{22}}{66} S_{3,3} - \frac{\sqrt{2}}{6} S_{3,1} + \frac{5\sqrt{231}}{462} S_{3,1} - \frac{\sqrt{30}}{15} S_{3,1} + \frac{\sqrt{105}}{70} S_{3,1} \right].$$

$$d_{-4} t_2 = N \frac{\sqrt{165}}{33} S_{3,-4}.$$

$$d_{-4} t_{-4} = -N \left[\frac{\sqrt{165}}{33} S_{3,4} - \frac{5\sqrt{154}}{462} S_{3,3} + \frac{\sqrt{2}}{3} S_{3,2} - \frac{\sqrt{42}}{14} S_{3,1} \right].$$

$$d_{-4} t_3 = N \left[\frac{5\sqrt{78}}{78} S_{3,-1} - \frac{\sqrt{385}}{462} S_{3,-1} + \frac{\sqrt{2}}{6} S_{3,-1} + \frac{3\sqrt{7}}{14} S_{3,-1} \right].$$

$$d_{-4} t_{-3} = -N \left[\frac{5\sqrt{78}}{78} S_{3,3} - \frac{\sqrt{385}}{462} S_{3,1} + \frac{\sqrt{2}}{6} S_{3,1} + \frac{3\sqrt{7}}{14} S_{3,1} \right].$$

$$t_4 t_4 = N \left[\frac{50\sqrt{26}}{429} S_{4,2} + \frac{3\sqrt{2}}{11} S_{4,2} + \frac{2\sqrt{10}}{15} S_{2,2} + \frac{\sqrt{2}}{2} S_{3,2} \right].$$

$$t_4 t_1 = N \left[\frac{50\sqrt{91}}{936} S_{4,1} + \frac{\sqrt{30}}{22} S_{4,1} + \frac{\sqrt{5}}{15} S_{4,1} \right].$$

$$t_4 t_{-1} = N \left[\frac{50\sqrt{91}}{936} S_{4,-1} + \frac{\sqrt{30}}{22} S_{4,-1} + \frac{\sqrt{5}}{15} S_{4,-1} \right].$$

$$t_4 t_2 = N \left[\frac{2\sqrt{91}}{39} S_{4,2} - \frac{\sqrt{6}}{22} S_{4,2} - \frac{\sqrt{2}}{3} S_{4,2} \right].$$

$$f_6 L_2 = N \left[\frac{2\sqrt{91}}{39} S_{k-2} - \frac{\sqrt{6}}{22} S_{k-1} - \frac{\sqrt{2}}{3} S_{k-3} \right].$$

$$f_6 f_5 = N \left[\frac{5\sqrt{546}}{429} S_{k,2} - \frac{3\sqrt{14}}{22} S_{k,3} \right].$$

$$f_6 L_3 = N \left[\frac{5\sqrt{546}}{429} S_{k-3} - \frac{3\sqrt{14}}{22} S_{k-1} \right].$$

$$f_4 f_1 = N \left[\frac{5\sqrt{1365}}{429} S_{k,2} + \frac{\sqrt{10}}{11} S_{k,3} + \frac{\sqrt{30}}{15} S_{k,2} - \frac{25\sqrt{26}}{286} S_{k,0} + \frac{\sqrt{2}}{22} S_{k,0} + \frac{\sqrt{10}}{10} S_{k,0} + \frac{\sqrt{2}}{2} S_{k,0} \right].$$

$$f_4 L_1 = N \left[\frac{5\sqrt{1365}}{429} S_{k-1} + \frac{\sqrt{10}}{11} S_{k-2} + \frac{\sqrt{30}}{15} S_{k-1} \right].$$

$$f_4 f_2 = N \left[\frac{5\sqrt{546}}{286} S_{k,2} + \frac{\sqrt{14}}{22} S_{k,3} - \frac{5\sqrt{1365}}{858} S_{k,1} + \frac{2\sqrt{2}}{11} S_{k,1} + \frac{\sqrt{3}}{6} S_{k,1} \right].$$

$$f_4 L_2 = N \left[\frac{5\sqrt{546}}{286} S_{k-3} + \frac{\sqrt{14}}{22} S_{k-1} - \frac{5\sqrt{1365}}{858} S_{k-1} + \frac{2\sqrt{2}}{11} S_{k-1} + \frac{\sqrt{3}}{6} S_{k-1} \right].$$

$$f_4 f_3 = N \left[\frac{5\sqrt{2730}}{858} S_{k,4} - \frac{\sqrt{42}}{22} S_{k,4} - \frac{5\sqrt{91}}{429} S_{k,2} + \frac{3\sqrt{6}}{22} S_{k,2} - \frac{\sqrt{2}}{6} S_{k,2} \right].$$

$$f_4 L_3 = N \left[\frac{5\sqrt{2730}}{858} S_{k-4} - \frac{\sqrt{42}}{22} S_{k-4} - \frac{5\sqrt{91}}{429} S_{k-1} + \frac{3\sqrt{6}}{22} S_{k-1} - \frac{\sqrt{2}}{6} S_{k-1} \right].$$

$$L_1 f_1 = -N \left[\frac{5\sqrt{546}}{429} S_{k,2} + \frac{\sqrt{10}}{11} S_{k,3} + \frac{\sqrt{30}}{15} S_{k,2} + \frac{25\sqrt{26}}{286} S_{k,0} - \frac{\sqrt{2}}{22} S_{k,0} - \frac{\sqrt{10}}{10} S_{k,0} - \frac{\sqrt{2}}{2} S_{k,0} \right].$$

$$L_1 f_2 = N \left[\frac{5\sqrt{546}}{286} S_{k-3} + \frac{\sqrt{14}}{22} S_{k-1} + \frac{5\sqrt{1365}}{858} S_{k-1} - \frac{2\sqrt{2}}{11} S_{k-1} - \frac{\sqrt{3}}{6} S_{k-1} \right].$$

$$L_1 f_3 = -N \left[\frac{5\sqrt{546}}{286} S_{k,2} + \frac{\sqrt{14}}{22} S_{k,3} + \frac{5\sqrt{1365}}{858} S_{k,1} - \frac{2\sqrt{2}}{11} S_{k,1} - \frac{\sqrt{3}}{6} S_{k,1} \right].$$

$$L_1 f_4 = N \left[\frac{5\sqrt{2730}}{858} S_{k-4} - \frac{\sqrt{42}}{22} S_{k-4} + \frac{5\sqrt{91}}{429} S_{k-1} - \frac{3\sqrt{6}}{22} S_{k-1} + \frac{\sqrt{2}}{6} S_{k-1} \right].$$

$$L_1 L_2 = -N \left[\frac{5\sqrt{2730}}{858} S_{k,4} - \frac{\sqrt{42}}{22} S_{k,4} + \frac{5\sqrt{91}}{429} S_{k,2} - \frac{3\sqrt{6}}{22} S_{k,2} + \frac{\sqrt{2}}{6} S_{k,2} \right].$$

$$f_1 f_5 = N \left[\frac{5\sqrt{182}}{143} S_{k,4} + \frac{\sqrt{70}}{22} S_{k,4} + \frac{5\sqrt{26}}{143} S_{k,0} - \frac{7\sqrt{2}}{22} S_{k,0} + \frac{\sqrt{2}}{2} S_{k,0} \right].$$

$$f_1 L_3 = N \left[\frac{5\sqrt{182}}{143} S_{k-1} + \frac{\sqrt{70}}{22} S_{k-4} \right].$$

$$t_4 t_4 = N \left[\frac{5 \sqrt{6006}}{858} s_{4,3} + \frac{5 \sqrt{91}}{858} s_{4,3} - \frac{\sqrt{30}}{22} s_{4,1} + \frac{\sqrt{5}}{6} s_{2,1} \right].$$

$$t_4 t_{-3} = N \left[\frac{5 \sqrt{6006}}{858} s_{4,-3} + \frac{5 \sqrt{91}}{858} s_{4,-3} - \frac{\sqrt{30}}{22} s_{4,-1} + \frac{\sqrt{5}}{6} s_{2,-1} \right].$$

$$t_{-3} t_{-3} = -N \left[\frac{5 \sqrt{182}}{143} s_{4,4} + \frac{\sqrt{70}}{22} s_{4,4} - \frac{5 \sqrt{26}}{143} s_{4,0} + \frac{7 \sqrt{2}}{22} s_{4,0} - \frac{\sqrt{2}}{2} s_{4,0} \right].$$

$$t_{-3} t_3 = N \left[\frac{5 \sqrt{6006}}{858} s_{4,-4} - \frac{5 \sqrt{91}}{858} s_{4,-4} + \frac{\sqrt{30}}{22} s_{4,-1} - \frac{\sqrt{5}}{6} s_{2,-1} \right].$$

$$t_{-3} t_{-3} = -N \left[\frac{5 \sqrt{6006}}{858} s_{4,4} - \frac{5 \sqrt{91}}{858} s_{4,4} + \frac{\sqrt{30}}{22} s_{4,1} - \frac{\sqrt{5}}{6} s_{2,1} \right].$$

$$t_4 t_3 = N \left[\frac{5 \sqrt{3003}}{429} s_{4,0} - \frac{5 \sqrt{26}}{858} s_{4,0} + \frac{3 \sqrt{2}}{22} s_{4,0} - \frac{\sqrt{10}}{6} s_{2,0} + \frac{\sqrt{2}}{2} s_{0,0} \right].$$

$$t_4 t_{-3} = N \left[\frac{5 \sqrt{3003}}{429} s_{4,-4} \right].$$

$$t_{-3} t_{-3} = -N \left[\frac{5 \sqrt{3003}}{429} s_{4,4} + \frac{5 \sqrt{26}}{858} s_{4,0} - \frac{3 \sqrt{2}}{22} s_{4,0} + \frac{\sqrt{10}}{6} s_{2,0} - \frac{\sqrt{2}}{2} s_{0,0} \right].$$

TABELLA III

Valori dei coefficienti della funzione $F(u, v)$

| L/L'M | α | β | β' |
|-------|--------------------------|---------------------------|----------------|
| 000 | 2 | 2 | 2 |
| 010 | $\frac{2\sqrt{3}}{3}$ | $\frac{2\sqrt{3}}{3}$ | 0 |
| 020 | $\frac{2\sqrt{5}}{5}$ | $\frac{2\sqrt{5}}{5}$ | 0 |
| 030 | $\frac{2\sqrt{7}}{7}$ | $\frac{2\sqrt{7}}{7}$ | 0 |
| 040 | $\frac{2}{3}$ | $\frac{2}{3}$ | 0 |
| 050 | $\frac{2\sqrt{11}}{11}$ | $\frac{2\sqrt{11}}{11}$ | 0 |
| 060 | $\frac{2\sqrt{13}}{13}$ | $\frac{2\sqrt{13}}{13}$ | 0 |
| 1'0 | $\frac{4}{3}$ | $-\frac{2}{3}$ | $-\frac{2}{3}$ |
| 120 | $\frac{2\sqrt{15}}{5}$ | $-\frac{4\sqrt{15}}{15}$ | 0 |
| 130 | $\frac{8\sqrt{21}}{21}$ | $-\frac{2\sqrt{21}}{7}$ | 0 |
| 140 | $\frac{10\sqrt{3}}{9}$ | $-\frac{8\sqrt{3}}{9}$ | 0 |
| 150 | $\frac{12\sqrt{33}}{33}$ | $-\frac{10\sqrt{33}}{33}$ | 0 |
| 160 | $\frac{14\sqrt{39}}{39}$ | $-\frac{4\sqrt{39}}{13}$ | 0 |

| | | | |
|-----|---------------------------|-----------------------------|-----------------|
| 220 | $\frac{12}{5}$ | $\frac{2}{5}$ | $\frac{2}{5}$ |
| 230 | $\frac{4\sqrt{35}}{7}$ | $\frac{6\sqrt{35}}{35}$ | 0 |
| 240 | $2\sqrt{5}$ | $\frac{4\sqrt{5}}{5}$ | 0 |
| 250 | $\frac{42\sqrt{55}}{55}$ | $\frac{4\sqrt{55}}{11}$ | 0 |
| 260 | $\frac{56\sqrt{65}}{65}$ | $\frac{6\sqrt{65}}{13}$ | 0 |
| 330 | $\frac{40}{7}$ | $-\frac{2}{7}$ | $-\frac{2}{7}$ |
| 340 | $\frac{10\sqrt{7}}{3}$ | $-\frac{8\sqrt{7}}{21}$ | 0 |
| 350 | $\frac{16\sqrt{77}}{11}$ | $-\frac{20\sqrt{77}}{77}$ | 0 |
| 360 | $\frac{24\sqrt{91}}{13}$ | $-\frac{40\sqrt{91}}{91}$ | 0 |
| 440 | $\frac{140}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ |
| 450 | $\frac{84\sqrt{11}}{11}$ | $\frac{10\sqrt{11}}{33}$ | 0 |
| 460 | $\frac{140\sqrt{13}}{13}$ | $\frac{10\sqrt{13}}{13}$ | 0 |
| 550 | $\frac{504}{11}$ | $-\frac{2}{11}$ | $-\frac{2}{11}$ |
| 560 | $\frac{84\sqrt{143}}{13}$ | $-\frac{12\sqrt{143}}{143}$ | 0 |
| 660 | $\frac{1848}{13}$ | $\frac{2}{13}$ | $\frac{2}{13}$ |
| 111 | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ |

| | | | |
|-----|---------------------------|---------------------------|----------------|
| 121 | $\frac{2\sqrt{5}}{5}$ | $\frac{2\sqrt{5}}{5}$ | 0 |
| 131 | $\frac{2\sqrt{14}}{7}$ | $\frac{2\sqrt{14}}{7}$ | 0 |
| 141 | $\frac{2\sqrt{30}}{9}$ | $\frac{2\sqrt{30}}{9}$ | 0 |
| 151 | $\frac{2\sqrt{55}}{11}$ | $\frac{2\sqrt{55}}{11}$ | 0 |
| 161 | $\frac{2\sqrt{91}}{13}$ | $\frac{2\sqrt{91}}{13}$ | 0 |
| 221 | $\frac{8}{5}$ | $-\frac{2}{5}$ | $-\frac{2}{5}$ |
| 231 | $\frac{4\sqrt{70}}{14}$ | $-\frac{4\sqrt{70}}{35}$ | 0 |
| 241 | $\frac{4\sqrt{6}}{3}$ | $-\frac{2\sqrt{6}}{3}$ | 0 |
| 251 | $\frac{14\sqrt{11}}{11}$ | $-\frac{8\sqrt{11}}{11}$ | 0 |
| 261 | $\frac{16\sqrt{455}}{65}$ | $-\frac{2\sqrt{455}}{13}$ | 0 |
| 331 | $\frac{30}{7}$ | $\frac{2}{7}$ | $\frac{2}{7}$ |
| 341 | $\frac{2\sqrt{105}}{3}$ | $\frac{2\sqrt{105}}{21}$ | 0 |
| 351 | $\frac{4\sqrt{770}}{11}$ | $\frac{6\sqrt{770}}{77}$ | 0 |
| 361 | $\frac{36\sqrt{26}}{13}$ | $\frac{10\sqrt{26}}{13}$ | 0 |
| 441 | $\frac{112}{9}$ | $-\frac{2}{9}$ | $-\frac{2}{9}$ |
| 451 | $\frac{28\sqrt{66}}{11}$ | $-\frac{4\sqrt{66}}{33}$ | 0 |

| | | | |
|-----|----------------------------|----------------------------|-----------------|
| 461 | $\frac{8\sqrt{2730}}{13}$ | $-\frac{2\sqrt{2730}}{39}$ | 0 |
| 551 | $\frac{420}{11}$ | $\frac{2}{11}$ | $\frac{2}{11}$ |
| 561 | $\frac{12\sqrt{5005}}{13}$ | $\frac{2\sqrt{5005}}{143}$ | 0 |
| 661 | $\frac{1584}{13}$ | $-\frac{2}{13}$ | $-\frac{2}{13}$ |
| 922 | $\frac{2}{5}$ | $\frac{2}{5}$ | $\frac{2}{5}$ |
| 232 | $\frac{2\sqrt{7}}{7}$ | $\frac{2\sqrt{7}}{7}$ | 0 |
| 242 | $\frac{2\sqrt{3}}{3}$ | $\frac{2\sqrt{3}}{3}$ | 0 |
| 252 | $\frac{2\sqrt{77}}{11}$ | $\frac{2\sqrt{77}}{11}$ | 0 |
| 262 | $\frac{2\sqrt{182}}{13}$ | $\frac{2\sqrt{182}}{13}$ | 0 |
| 332 | $\frac{12}{7}$ | $-\frac{2}{7}$ | $-\frac{2}{7}$ |
| 342 | $\frac{2\sqrt{21}}{3}$ | $-\frac{4\sqrt{21}}{21}$ | 0 |
| 352 | $\frac{16\sqrt{11}}{11}$ | $-\frac{6\sqrt{11}}{11}$ | 0 |
| 362 | $\frac{18\sqrt{26}}{13}$ | $-\frac{8\sqrt{26}}{13}$ | 0 |
| 442 | $\frac{56}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ |
| 452 | $\frac{8\sqrt{231}}{11}$ | $\frac{2\sqrt{231}}{33}$ | 0 |
| 462 | $\frac{10\sqrt{546}}{13}$ | $\frac{4\sqrt{546}}{39}$ | 0 |

| | | | |
|-----|---------------------------|----------------------------|-----------------|
| 552 | $\frac{240}{11}$ | $-\frac{2}{11}$ | $-\frac{2}{11}$ |
| 562 | $\frac{36\sqrt{286}}{13}$ | $-\frac{8\sqrt{286}}{143}$ | 0 |
| 662 | $\frac{990}{13}$ | $\frac{2}{13}$ | $\frac{2}{13}$ |
| 333 | $\frac{2}{7}$ | $\frac{2}{7}$ | $\frac{2}{7}$ |
| 343 | $\frac{2}{3}$ | $\frac{2}{3}$ | 0 |
| 353 | $\frac{4\sqrt{11}}{11}$ | $\frac{4\sqrt{11}}{11}$ | 0 |
| 363 | $\frac{4\sqrt{39}}{13}$ | $\frac{4\sqrt{39}}{13}$ | 0 |
| 443 | $\frac{16}{9}$ | $-\frac{2}{9}$ | $-\frac{2}{9}$ |
| 453 | $\frac{12\sqrt{11}}{11}$ | $-\frac{8\sqrt{11}}{33}$ | 0 |
| 463 | $\frac{40\sqrt{39}}{39}$ | $-\frac{4\sqrt{39}}{13}$ | 0 |
| 553 | $\frac{90}{11}$ | $\frac{2}{11}$ | $\frac{2}{11}$ |
| 563 | $\frac{10\sqrt{429}}{13}$ | $\frac{6\sqrt{429}}{143}$ | 0 |
| 663 | $\frac{440}{13}$ | $-\frac{2}{13}$ | $-\frac{2}{13}$ |
| 444 | $\frac{2}{9}$ | $\frac{2}{9}$ | $\frac{2}{9}$ |
| 454 | $\frac{2\sqrt{11}}{11}$ | $\frac{2\sqrt{11}}{11}$ | 0 |
| 464 | $\frac{2\sqrt{65}}{13}$ | $\frac{2\sqrt{65}}{13}$ | 0 |

| | | | |
|-----|--------------------------|----------------------------|-----------------|
| 554 | $\frac{20}{11}$ | $-\frac{2}{11}$ | $-\frac{2}{11}$ |
| 564 | $\frac{2\sqrt{715}}{13}$ | $-\frac{4\sqrt{715}}{143}$ | 0 |
| 664 | $\frac{132}{13}$ | $\frac{2}{13}$ | $\frac{2}{13}$ |
| 565 | $\frac{2\sqrt{13}}{13}$ | $\frac{2\sqrt{13}}{13}$ | 0 |
| 665 | $\frac{24}{13}$ | $-\frac{2}{13}$ | $-\frac{2}{13}$ |
| 666 | $\frac{2}{13}$ | $\frac{2}{13}$ | $\frac{2}{13}$ |

TABELLA IV

Decomposizione degli integrali coulombiani fra orbitali s, p, d e f
in integrali elementari

$$(ss | ss) = \frac{1}{2} J (000)$$

$$|s p_0) = \frac{1}{2} J (010)$$

$$|s d_0) = \frac{1}{2} J (020)$$

$$|s f_0) = \frac{1}{2} J (030)$$

$$|p_0 p_0) = \frac{1}{2} J (000) + \frac{\sqrt{5}}{5} J (020)$$

$$|p_0 d_0) = \frac{\sqrt{5}}{5} J (010) + \frac{3\sqrt{105}}{70} J (030)$$

$$|p_0 f_0) = \frac{3\sqrt{105}}{70} J (020) + \frac{2\sqrt{21}}{21} J (040)$$

$$|p_1 p_1) = \frac{1}{2} J (000) - \frac{\sqrt{5}}{10} J (020)$$

$$|p_1 d_1) = \frac{\sqrt{15}}{10} J (010) - \frac{3\sqrt{35}}{70} J (030)$$

$$|p_1 f_1) = \frac{3\sqrt{70}}{70} J (020) - \frac{\sqrt{14}}{14} J (040)$$

$$|d_0 d_0) = \frac{1}{2} J (000) + \frac{\sqrt{5}}{4} J (020) + \frac{3}{7} J (040)$$

$$|d_0 f_0) = \frac{3\sqrt{105}}{70} J (010) + \frac{2\sqrt{5}}{15} J (030)$$

$$|d_1 d_1) = \frac{1}{2} J (000) + \frac{5\sqrt{5}}{70} J (020) - \frac{2}{7} J (040)$$

$$|d_1 f_1) = \frac{\sqrt{210}}{35} J (010) + \frac{\sqrt{10}}{30} J (030) - \frac{5\sqrt{770}}{462} J (050)$$

$$|d_4 d_4\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{7} J(020) + \frac{1}{14} J(040)$$

$$|d_4 f_4\rangle = \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050)$$

$$|f_4 f_4\rangle = \frac{1}{2} J(010) + \frac{2\sqrt{5}}{15} J(020) + \frac{3}{11} J(040) + \frac{50\sqrt{13}}{429} J(060)$$

$$|f_4 f_1\rangle = \frac{1}{2} J(000) + \frac{\sqrt{5}}{10} J(020) + \frac{1}{22} J(040) - \frac{25\sqrt{13}}{286} J(060)$$

$$|f_4 f_3\rangle = \frac{1}{2} J(000) - \frac{7}{22} J(040) + \frac{5\sqrt{13}}{143} J(060)$$

$$|f_4 f_2\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{6} J(020) + \frac{3}{22} J(040) - \frac{5\sqrt{13}}{858} J(060)$$

$$|s p_0\rangle |s p_0\rangle = \frac{1}{2} J(110)$$

$$|s d_0\rangle = \frac{1}{2} J(120)$$

$$|s f_0\rangle = \frac{1}{2} J(130)$$

$$|p_0 p_0\rangle = \frac{1}{2} J(010) + \frac{\sqrt{5}}{5} J(120)$$

$$|p_0 d_0\rangle = \frac{\sqrt{5}}{5} J(110) + \frac{3\sqrt{105}}{70} J(130)$$

$$|p_0 f_0\rangle = \frac{3\sqrt{105}}{70} J(120) + \frac{2\sqrt{21}}{21} J(140)$$

$$|p_1 p_1\rangle = \frac{1}{2} J(010) - \frac{\sqrt{5}}{10} J(120)$$

$$|p_1 d_1\rangle = \frac{\sqrt{15}}{10} J(110) - \frac{3\sqrt{35}}{70} J(130)$$

$$|p_1 f_1\rangle = \frac{3\sqrt{70}}{70} J(120) - \frac{\sqrt{14}}{14} J(140)$$

$$|d_0 d_0\rangle = \frac{1}{2} J(010) + \frac{\sqrt{5}}{7} J(120) + \frac{3}{7} J(140)$$

$$|d_4 f_4\rangle = \frac{3\sqrt{105}}{70} J(110) + \frac{2\sqrt{5}}{15} J(130)$$

$$|d_4 d_4\rangle = \frac{1}{2} J(010) + \frac{\sqrt{5}}{14} J(120) - \frac{2}{7} J(140)$$

$$|d_4 f_1\rangle = \frac{\sqrt{210}}{35} J(110) + \frac{\sqrt{10}}{30} J(130) - \frac{5\sqrt{770}}{462} J(150)$$

$$|d_4 d_2\rangle = \frac{1}{2} J(010) - \frac{\sqrt{5}}{7} J(120) + \frac{1}{14} J(140)$$

$$|d_4 f_2\rangle = \frac{\sqrt{21}}{14} J(110) - \frac{1}{3} J(130) + \frac{5\sqrt{77}}{462} J(150)$$

$$|f_4 f_4\rangle = \frac{1}{2} J(010) + \frac{2\sqrt{5}}{15} J(120) + \frac{3}{11} J(140) + \frac{50\sqrt{13}}{420} J(160)$$

$$|f_4 f_1\rangle = \frac{1}{2} J(010) + \frac{\sqrt{5}}{10} J(120) + \frac{1}{22} J(140) - \frac{25\sqrt{13}}{286} J(160)$$

$$|f_4 f_2\rangle = \frac{1}{2} J(010) - \frac{7}{22} J(140) + \frac{5\sqrt{13}}{143} J(160)$$

$$|f_4 f_3\rangle = \frac{1}{2} J(010) - \frac{\sqrt{5}}{6} J(120) + \frac{3}{22} J(140) - \frac{5\sqrt{13}}{858} J(160)$$

$$|s p_1 | s p_1\rangle = \frac{1}{2} J(111)$$

$$|s d_1\rangle = \frac{1}{2} J(121)$$

$$|s f_1\rangle = \frac{1}{2} J(131)$$

$$|p_0 p_1\rangle = \frac{\sqrt{15}}{10} J(121)$$

$$|p_0 d_1\rangle = \frac{\sqrt{15}}{10} J(111) + \frac{\sqrt{210}}{35} J(131)$$

$$|p_0 f_1\rangle = \frac{\sqrt{210}}{35} J(121) + \frac{\sqrt{35}}{14} J(141)$$

$$|p_1 d_0\rangle = -\frac{\sqrt{5}}{10} J(111) + \frac{3\sqrt{70}}{70} J(131)$$

$$p_1 d_1 = \frac{\sqrt{15}}{10} J (111) - \frac{\sqrt{210}}{140} J (131)$$

$$| p_1 f_0 = -\frac{3\sqrt{35}}{70} J (121) + \frac{\sqrt{210}}{42} J (141)$$

$$| p_1 f_2 = \frac{\sqrt{21}}{14} J (121) - \frac{\sqrt{14}}{28} J (141)$$

$$| d_0 d_1 = \frac{\sqrt{5}}{14} J (121) + \frac{\sqrt{30}}{14} J (141)$$

$$| d_0 f_1 = \frac{3\sqrt{70}}{70} J (111) + \frac{\sqrt{5}}{10} J (131) + \frac{5\sqrt{154}}{154} J (151)$$

$$| d_1 d_2 = \frac{\sqrt{15}}{14} J (121) - \frac{\sqrt{10}}{28} J (141)$$

$$| d_1 f_0 = -\frac{3\sqrt{35}}{70} J (111) + \frac{\sqrt{10}}{30} J (131) + \frac{10\sqrt{77}}{231} J (151)$$

$$| d_1 f_2 = \frac{\sqrt{21}}{14} J (111) + \frac{\sqrt{6}}{12} J (131) - \frac{\sqrt{1155}}{231} J (151)$$

$$| d_2 f_1 = -\frac{\sqrt{210}}{140} J (111) + \frac{\sqrt{15}}{15} J (131) - \frac{5\sqrt{462}}{924} J (151)$$

$$| d_2 f_2 = \frac{3\sqrt{14}}{28} J (111) - \frac{1}{6} J (131) + \frac{\sqrt{770}}{924} J (151)$$

$$| f_0 f_1 = \frac{\sqrt{10}}{30} J (121) + \frac{\sqrt{15}}{22} J (141) + \frac{25\sqrt{182}}{936} J (161)$$

$$| f_1 f_2 = \frac{\sqrt{6}}{12} J (121) + \frac{2}{11} J (141) - \frac{5\sqrt{2730}}{1716} J (161)$$

$$| f_2 f_3 = \frac{\sqrt{10}}{12} J (121) - \frac{\sqrt{15}}{22} J (141) + \frac{5\sqrt{182}}{1716} J (161)$$

$$(* d_0 | s d_0 = \frac{1}{2} J (220)$$

$$| s f_0 = \frac{1}{2} J (230)$$

$$| p_0 p_0 = \frac{1}{2} J (020) + \frac{\sqrt{5}}{5} J (220)$$

$$|p_0 d_0\rangle = \frac{\sqrt{5}}{5} J(120) + \frac{3\sqrt{105}}{70} J(230)$$

$$|p_0 f_0\rangle = \frac{3\sqrt{105}}{70} J(220) + \frac{2\sqrt{21}}{21} J(240)$$

$$|p_1 p_1\rangle = \frac{1}{2} J(020) - \frac{\sqrt{5}}{10} J(220)$$

$$|p_1 d_1\rangle = \frac{\sqrt{15}}{10} J(120) - \frac{3\sqrt{35}}{70} J(230)$$

$$|p_1 f_1\rangle = \frac{3\sqrt{70}}{70} J(220) - \frac{\sqrt{14}}{14} J(240)$$

$$|d_0 d_0\rangle = \frac{1}{2} J(020) + \frac{\sqrt{5}}{7} J(220) + \frac{3}{7} J(240)$$

$$|d_0 f_0\rangle = \frac{3\sqrt{105}}{70} J(120) + \frac{2\sqrt{5}}{15} J(230)$$

$$|d_1 d_1\rangle = \frac{1}{2} J(020) + \frac{5\sqrt{5}}{70} J(220) - \frac{2}{7} J(240)$$

$$|d_1 f_1\rangle = \frac{\sqrt{210}}{35} J(120) + \frac{\sqrt{10}}{30} J(230) - \frac{5\sqrt{770}}{462} J(250)$$

$$|d_2 d_2\rangle = \frac{1}{2} J(020) - \frac{\sqrt{5}}{7} J(220) + \frac{1}{14} J(240)$$

$$|d_2 f_2\rangle = \frac{\sqrt{21}}{14} J(120) - \frac{1}{3} J(230) + \frac{5\sqrt{77}}{462} J(250)$$

$$|f_0 f_0\rangle = \frac{1}{2} J(020) + \frac{2\sqrt{5}}{15} J(220) + \frac{3}{11} J(240) + \frac{50\sqrt{13}}{429} J(260)$$

$$|f_1 f_1\rangle = \frac{1}{2} J(020) + \frac{\sqrt{5}}{10} J(220) + \frac{1}{22} J(240) - \frac{25\sqrt{13}}{286} J(260)$$

$$|f_2 f_2\rangle = \frac{1}{2} J(020) - \frac{7}{22} J(240) + \frac{5\sqrt{13}}{143} J(260)$$

$$|f_3 f_3\rangle = \frac{1}{2} J(020) - \frac{\sqrt{5}}{6} J(220) + \frac{3}{22} J(240) - \frac{5\sqrt{13}}{858} J(260)$$

$$(* d_4 | s d_4) = \frac{1}{2} J(221)$$

$$[s f_1] = \frac{1}{2} J \text{ (231)}$$

$$[p_6 p_7] = \frac{\sqrt{15}}{10} J \text{ (221)}$$

$$[p_6 d_1] = \frac{\sqrt{15}}{10} J \text{ (121)} + \frac{\sqrt{210}}{35} J \text{ (231)}$$

$$[p_6 f_1] = \frac{\sqrt{210}}{35} J \text{ (221)} + \frac{\sqrt{35}}{14} J \text{ (241)}$$

$$[p_7 d_6] = -\frac{\sqrt{5}}{10} J \text{ (121)} + \frac{3\sqrt{70}}{70} J \text{ (231)}$$

$$[p_7 d_7] = \frac{\sqrt{15}}{10} J \text{ (121)} - \frac{\sqrt{210}}{140} J \text{ (231)}$$

$$[p_7 f_6] = -\frac{3\sqrt{35}}{70} J \text{ (221)} + \frac{\sqrt{210}}{42} J \text{ (241)}$$

$$[p_7 f_7] = \frac{\sqrt{21}}{14} J \text{ (221)} - \frac{\sqrt{14}}{28} J \text{ (241)}$$

$$[d_6 d_1] = \frac{\sqrt{5}}{14} J \text{ (221)} + \frac{\sqrt{30}}{14} J \text{ (241)}$$

$$[d_6 f_1] = \frac{3\sqrt{70}}{70} J \text{ (121)} + \frac{\sqrt{5}}{10} J \text{ (231)} + \frac{5\sqrt{154}}{154} J \text{ (251)}$$

$$[d_7 d_2] = \frac{\sqrt{15}}{14} J \text{ (221)} - \frac{\sqrt{10}}{28} J \text{ (241)}$$

$$[d_7 f_6] = -\frac{3\sqrt{35}}{70} J \text{ (121)} + \frac{\sqrt{10}}{30} J \text{ (231)} + \frac{10\sqrt{77}}{231} J \text{ (251)}$$

$$[d_7 f_7] = \frac{\sqrt{21}}{14} J \text{ (121)} + \frac{\sqrt{6}}{12} J \text{ (231)} - \frac{\sqrt{1155}}{231} J \text{ (251)}$$

$$[d_2 f_1] = -\frac{\sqrt{210}}{140} J \text{ (121)} + \frac{\sqrt{15}}{15} J \text{ (231)} - \frac{5\sqrt{462}}{924} J \text{ (251)}$$

$$[d_2 f_6] = \frac{3\sqrt{14}}{28} J \text{ (121)} - \frac{1}{6} J \text{ (231)} + \frac{\sqrt{770}}{924} J \text{ (251)}$$

$$[f_6 f_1] = \frac{\sqrt{10}}{30} J \text{ (221)} + \frac{\sqrt{15}}{22} J \text{ (241)} + \frac{25\sqrt{182}}{936} J \text{ (261)}$$

$$|f_1 f_2\rangle = \frac{\sqrt{6}}{12} J(221) + \frac{2}{11} J(241) - \frac{5\sqrt{2730}}{1716} J(261)$$

$$|f_2 f_3\rangle = \frac{\sqrt{10}}{12} J(221) - \frac{\sqrt{15}}{22} J(241) + \frac{5\sqrt{182}}{1716} J(261)$$

$$|s d_2\rangle |s d_2\rangle = \frac{1}{2} J(222)$$

$$|s f_2\rangle = \frac{1}{2} J(232)$$

$$|p_0 d_2\rangle = \frac{\sqrt{21}}{14} J(232)$$

$$|p_0 f_2\rangle = \frac{\sqrt{21}}{14} J(232) + \frac{\sqrt{7}}{7} J(242)$$

$$|p_1 p_1\rangle = \frac{\sqrt{15}}{10} J(222)$$

$$|p_1 d_1\rangle = \frac{\sqrt{21}}{14} J(232)$$

$$|p_1 f_1\rangle = \frac{\sqrt{70}}{28} J(242) - \frac{\sqrt{210}}{140} J(222)$$

$$|p_1 f_2\rangle = \frac{3\sqrt{14}}{28} J(222) - \frac{\sqrt{42}}{84} J(242)$$

$$|d_2 d_2\rangle = -\frac{\sqrt{5}}{7} J(222) + \frac{\sqrt{15}}{14} J(242)$$

$$|d_2 f_2\rangle = \frac{\sqrt{55}}{22} J(252)$$

$$|d_2 d_1\rangle = \frac{\sqrt{15}}{14} J(222) + \frac{\sqrt{5}}{7} J(242)$$

$$|d_1 f_1\rangle = \frac{\sqrt{6}}{12} J(232) + \frac{5\sqrt{66}}{132} J(252)$$

$$|d_1 f_2\rangle = \frac{\sqrt{10}}{12} J(232) - \frac{\sqrt{110}}{132} J(252)$$

$$|d_2 d_0\rangle = -\frac{1}{3} J(232) + \frac{5\sqrt{11}}{66} J(252)$$

$$|f_6 f_2| = -\frac{1}{3} J (222) - \frac{\sqrt{3}}{22} J (242) + \frac{\sqrt{182}}{39} J (262)$$

$$|f_4 f_1| = \frac{\sqrt{15}}{15} J (222) + \frac{\sqrt{5}}{11} J (242) + \frac{5\sqrt{2730}}{858} J (262)$$

$$|f_4 f_3| = -\frac{1}{6} J (222) + \frac{3\sqrt{3}}{22} J (242) - \frac{5\sqrt{182}}{858} J (262)$$

$$|s f_6| = \frac{1}{2} J (330)$$

$$|P_6 P_6| = \frac{1}{2} J (030) + \frac{\sqrt{5}}{5} J (230)$$

$$|P_6 d_6| = \frac{\sqrt{5}}{5} J (130) + \frac{3\sqrt{105}}{70} J (330)$$

$$|P_6 f_6| = \frac{3\sqrt{105}}{70} J (230) + \frac{2\sqrt{21}}{21} J (310)$$

$$|P_1 P_1| = \frac{1}{2} J (030) - \frac{\sqrt{5}}{10} J (230)$$

$$|P_1 d_1| = \frac{\sqrt{15}}{10} J (130) - \frac{3\sqrt{35}}{70} J (330)$$

$$|P_1 f_1| = \frac{3\sqrt{70}}{70} J (230) - \frac{\sqrt{14}}{14} J (340)$$

$$|d_6 d_6| = \frac{1}{2} J (030) + \frac{\sqrt{5}}{7} J (230) + \frac{3}{7} J (310)$$

$$|d_6 f_6| = \frac{3\sqrt{105}}{70} J (130) + \frac{2\sqrt{5}}{15} J (330)$$

$$|d_4 d_4| = \frac{1}{2} J (030) + \frac{\sqrt{5}}{14} J (230) - \frac{2}{7} J (310)$$

$$|d_4 f_4| = \frac{\sqrt{210}}{35} J (130) + \frac{\sqrt{10}}{30} J (330) - \frac{5\sqrt{770}}{462} J (350)$$

$$|d_2 d_2| = \frac{1}{2} J (030) - \frac{\sqrt{5}}{7} J (230) + \frac{1}{14} J (310)$$

$$|d_2 f_2| = \frac{\sqrt{21}}{14} J (130) - \frac{1}{3} J (330) + \frac{5\sqrt{77}}{462} J (350)$$

$$|f_0 f_0\rangle = \frac{1}{2} J (030) + \frac{2\sqrt{5}}{15} J (230) + \frac{3}{11} J (340) + \frac{50\sqrt{13}}{429} J (360)$$

$$|f_1 f_1\rangle = \frac{1}{2} J (030) + \frac{\sqrt{5}}{10} J (230) + \frac{1}{22} J (340) - \frac{25\sqrt{13}}{286} J (360)$$

$$|f_2 f_2\rangle = \frac{1}{2} J (030) - \frac{7}{22} J (340) + \frac{5\sqrt{13}}{143} J (360)$$

$$|f_3 f_3\rangle = \frac{1}{2} J (030) - \frac{\sqrt{5}}{6} J (230) + \frac{3}{22} J (340) - \frac{5\sqrt{13}}{858} J (360)$$

$$({}^8\bar{7}_4 | s f_1) = \frac{1}{2} J (331)$$

$$|p_0 p_0\rangle = \frac{\sqrt{15}}{10} J (231)$$

$$|p_0 d_1\rangle = \frac{\sqrt{15}}{10} J (131) + \frac{\sqrt{210}}{35} J (331)$$

$$|p_0 f_2\rangle = \frac{\sqrt{210}}{35} J (231) + \frac{\sqrt{35}}{14} J (341)$$

$$|p_1 d_0\rangle = -\frac{\sqrt{5}}{10} J (131) + \frac{3\sqrt{70}}{70} J (331)$$

$$|p_1 d_2\rangle = \frac{\sqrt{15}}{10} J (131) - \frac{\sqrt{210}}{140} J (331)$$

$$|p_1 f_0\rangle = -\frac{3\sqrt{35}}{70} J (231) + \frac{\sqrt{210}}{42} J (341)$$

$$|p_1 f_2\rangle = \frac{\sqrt{21}}{14} J (231) - \frac{\sqrt{14}}{28} J (341)$$

$$|d_0 d_1\rangle = \frac{\sqrt{5}}{14} J (231) + \frac{\sqrt{30}}{14} J (341)$$

$$|d_0 f_1\rangle = \frac{3\sqrt{70}}{70} J (131) + \frac{\sqrt{5}}{10} J (331) + \frac{5\sqrt{151}}{154} J (351)$$

$$|d_1 d_2\rangle = \frac{\sqrt{15}}{14} J (231) - \frac{\sqrt{10}}{28} J (341)$$

$$|d_1 f_0\rangle = -\frac{3\sqrt{35}}{70} J (131) + \frac{\sqrt{10}}{30} J (331) + \frac{10\sqrt{77}}{231} J (351)$$

$$|d_1 f_1) = \frac{\sqrt{21}}{14} J (331) + \frac{\sqrt{6}}{12} J (331) - \frac{\sqrt{1155}}{231} J (351)$$

$$|d_2 f_1) = -\frac{\sqrt{210}}{140} J (331) + \frac{\sqrt{15}}{15} J (331) - \frac{5\sqrt{462}}{924} J (351)$$

$$|d_3 f_1) = \frac{3\sqrt{14}}{28} J (331) - \frac{1}{6} J (331) + \frac{\sqrt{770}}{924} J (351)$$

$$|f_4 f_1) = \frac{\sqrt{10}}{30} J (231) + \frac{\sqrt{15}}{22} J (341) + \frac{25\sqrt{182}}{936} J (361)$$

$$|f_1 f_1) = \frac{\sqrt{6}}{12} J (231) + \frac{2}{11} J (341) - \frac{5\sqrt{2730}}{1716} J (361)$$

$$|f_2 f_1) = \frac{\sqrt{10}}{12} J (231) - \frac{\sqrt{15}}{22} J (341) + \frac{5\sqrt{182}}{1716} J (361)$$

$$({}^6 f_2 | s f_2) = \frac{1}{2} J (332)$$

$$|p_0 d_1) = \frac{\sqrt{21}}{14} J (332)$$

$$|p_0 f_1) = \frac{\sqrt{21}}{14} J (232) + \frac{\sqrt{7}}{7} J (342)$$

$$|p_1 p_1) = \frac{\sqrt{15}}{10} J (232)$$

$$|p_1 d_1) = \frac{\sqrt{21}}{14} J (332)$$

$$|p_1 f_1) = -\frac{\sqrt{210}}{140} J (232) + \frac{\sqrt{70}}{28} J (342)$$

$$|p_1 f_2) = \frac{3\sqrt{14}}{28} J (232) - \frac{\sqrt{42}}{84} J (342)$$

$$|d_0 d_1) = -\frac{\sqrt{5}}{7} J (232) + \frac{\sqrt{15}}{14} J (342)$$

$$|d_0 f_1) = \frac{\sqrt{55}}{22} J (352)$$

$$|d_1 d_1) = \frac{\sqrt{15}}{14} J (232) + \frac{\sqrt{5}}{7} J (342)$$

$$|d_1 f_1) = \frac{\sqrt{6}}{12} J (332) + \frac{5\sqrt{66}}{132} J (352)$$

$$|d_1 f_2) = \frac{\sqrt{10}}{12} J (332) - \frac{\sqrt{110}}{132} J (352)$$

$$|d_2 f_0) = -\frac{1}{3} J (332) + \frac{5\sqrt{11}}{66} J (352)$$

$$|f_0 f_2) = -\frac{1}{3} J (232) - \frac{\sqrt{3}}{22} J (342) + \frac{\sqrt{182}}{39} J (362)$$

$$|f_1 f_1) = \frac{\sqrt{15}}{15} J (232) + \frac{\sqrt{5}}{11} J (342) + \frac{5\sqrt{2730}}{858} J (362)$$

$$|f_1 f_2) = -\frac{1}{6} J (232) + \frac{3\sqrt{3}}{22} J (342) - \frac{5\sqrt{182}}{858} J (362)$$

$$(\textcircled{s} f_0 | s f_0) = \frac{1}{2} J (333)$$

$$|p_0 f_0) = \frac{\sqrt{3}}{6} J (343)$$

$$|p_1 d_2) = \frac{3\sqrt{14}}{28} J (333)$$

$$|p_1 f_2) = \frac{\sqrt{2}}{4} J (343)$$

$$|d_0 f_2) = -\frac{\sqrt{5}}{6} J (333) + \frac{\sqrt{55}}{33} J (353)$$

$$|d_1 d_2) = \frac{\sqrt{70}}{28} J (343)$$

$$|d_1 f_2) = \frac{\sqrt{10}}{12} J (333) + \frac{\sqrt{110}}{33} J (353)$$

$$|d_2 f_1) = -\frac{1}{6} J (333) + \frac{5\sqrt{11}}{66} J (353)$$

$$|f_0 f_2) = -\frac{3\sqrt{7}}{22} J (343) + \frac{5\sqrt{273}}{429} J (363)$$

$$|f_1 f_2) = \frac{\sqrt{7}}{22} J (343) + \frac{5\sqrt{273}}{286} J (363)$$

$$\begin{aligned}
 (P_0 P_0 | P_0 P_0) &= \frac{1}{2} J (000) + \frac{2\sqrt{5}}{5} J (020) + \frac{2}{5} J (220) \\
 | P_0 d_0) &= \frac{\sqrt{5}}{5} J (010) + \frac{3\sqrt{105}}{70} J (030) + \frac{2}{5} J (120) + \frac{3\sqrt{21}}{35} J (230) \\
 | P_0 f_0) &= \frac{4\sqrt{105}}{105} J (240) + \frac{3\sqrt{21}}{35} J (220) + \frac{2\sqrt{21}}{21} J (040) + \frac{3\sqrt{105}}{70} J (020) \\
 | P_1 P_1) &= \frac{1}{2} J (000) + \frac{\sqrt{5}}{10} J (020) - \frac{1}{5} J (220) \\
 | P_1 d_1) &= \frac{\sqrt{15}}{10} J (010) - \frac{3\sqrt{35}}{70} J (030) + \frac{\sqrt{3}}{5} J (120) - \frac{3\sqrt{7}}{35} J (230) \\
 | P_1 f_1) &= -\frac{\sqrt{70}}{35} J (240) + \frac{3\sqrt{14}}{35} J (220) - \frac{\sqrt{14}}{14} J (040) + \frac{3\sqrt{70}}{70} J (020) \\
 | d_4 d_4) &= \frac{1}{2} J (000) + \frac{12\sqrt{5}}{35} J (020) + \frac{3}{7} J (040) + \frac{2}{7} J (220) + \frac{6\sqrt{5}}{35} J (240) \\
 | d_4 f_4) &= \frac{10\sqrt{77}}{231} J (250) + \frac{4}{15} J (230) + \frac{3\sqrt{21}}{35} J (120) + \frac{5\sqrt{385}}{231} J (050) + \\
 &\quad + \frac{2\sqrt{5}}{15} J (030) + \frac{3\sqrt{105}}{70} J (010) \\
 | d_1 d_1) &= \frac{1}{2} J (000) + \frac{19\sqrt{5}}{70} J (020) - \frac{2}{7} J (040) + \frac{1}{7} J (220) - \frac{4\sqrt{5}}{35} J (240) \\
 | d_1 f_1) &= -\frac{5\sqrt{154}}{231} J (250) + \frac{\sqrt{2}}{15} J (230) + \frac{2\sqrt{42}}{35} J (120) - \frac{5\sqrt{770}}{462} J (050) + \\
 &\quad + \frac{\sqrt{10}}{30} J (030) + \frac{\sqrt{210}}{35} J (010) \\
 | d_2 d_2) &= \frac{1}{2} J (000) + \frac{2\sqrt{5}}{35} J (020) - \frac{2}{7} J (220) + \frac{\sqrt{5}}{35} J (240) + \frac{1}{14} J (040) \\
 | d_2 f_2) &= \frac{\sqrt{385}}{231} J (250) - \frac{2\sqrt{5}}{15} J (230) + \frac{\sqrt{105}}{35} J (120) + \frac{5\sqrt{77}}{462} J (050) - \\
 &\quad - \frac{1}{3} J (030) + \frac{\sqrt{21}}{14} J (010) \\
 | f_4 f_4) &= \frac{20\sqrt{65}}{429} J (260) + \frac{6\sqrt{5}}{55} J (240) + \frac{4}{15} J (220) + \frac{\sqrt{5}}{3} J (020) + \\
 &\quad + \frac{50\sqrt{13}}{429} J (060) + \frac{3}{11} J (040) + \frac{1}{2} J (000)
 \end{aligned}$$

$$|f_1 f_1\rangle = -\frac{5\sqrt{65}}{143} J(260) + \frac{\sqrt{5}}{55} J(240) + \frac{1}{5} J(220) + \frac{3\sqrt{5}}{10} J(020) - \\ - \frac{25\sqrt{13}}{286} J(060) + \frac{1}{22} J(040) + \frac{1}{2} J(000)$$

$$|f_2 f_2\rangle = \frac{2\sqrt{65}}{143} J(260) - \frac{7\sqrt{5}}{55} J(240) + \frac{\sqrt{5}}{5} J(020) + \frac{5\sqrt{13}}{143} J(060) - \\ - \frac{7}{22} J(040) + \frac{1}{2} J(000)$$

$$|f_3 f_3\rangle = -\frac{\sqrt{65}}{429} J(260) + \frac{3\sqrt{5}}{55} J(240) - \frac{1}{3} J(220) + \frac{\sqrt{5}}{30} J(020) - \\ - \frac{5\sqrt{13}}{858} J(060) + \frac{3}{22} J(040) + \frac{1}{2} J(000)$$

$$(P_0 P_1 | P_0 P_1) = \frac{3}{10} J(221)$$

$$|P_0 d_1\rangle = \frac{3}{10} J(121) + \frac{3\sqrt{14}}{35} J(231)$$

$$|P_0 f_1\rangle = \frac{\sqrt{21}}{14} J(241) + \frac{3\sqrt{14}}{35} J(221)$$

$$|P_1 d_0\rangle = -\frac{\sqrt{3}}{10} J(121) + \frac{3\sqrt{42}}{70} J(231)$$

$$|P_1 d_2\rangle = \frac{3}{10} J(121) - \frac{3\sqrt{14}}{140} J(231)$$

$$|P_1 f_0\rangle = \frac{\sqrt{14}}{14} J(241) - \frac{3\sqrt{21}}{70} J(221)$$

$$|P_1 f_2\rangle = -\frac{\sqrt{210}}{140} J(241) + \frac{3\sqrt{35}}{70} J(221)$$

$$|d_0 d_1\rangle = \frac{\sqrt{3}}{14} J(221) + \frac{3\sqrt{2}}{14} J(241)$$

$$|d_0 f_1\rangle = \frac{\sqrt{2310}}{154} J(251) + \frac{\sqrt{3}}{10} J(231) + \frac{3\sqrt{42}}{70} J(121)$$

$$|d_1 d_2\rangle = \frac{3}{14} J(221) - \frac{\sqrt{6}}{28} J(241)$$

$$[d_1 f_6] = \frac{2\sqrt{1155}}{231} J(251) + \frac{\sqrt{6}}{30} J(231) - \frac{3\sqrt{21}}{70} J(121)$$

$$[d_1 f_5] = -\frac{\sqrt{77}}{77} J(251) + \frac{\sqrt{10}}{20} J(231) + \frac{3\sqrt{35}}{70} J(121)$$

$$[d_2 f_1] = -\frac{\sqrt{770}}{308} J(251) + \frac{1}{5} J(231) - \frac{3\sqrt{14}}{140} J(121)$$

$$[d_2 f_2] = \frac{\sqrt{462}}{924} J(251) - \frac{\sqrt{15}}{30} J(231) + \frac{3\sqrt{210}}{140} J(121)$$

$$[f_3 f_1] = \frac{5\sqrt{2730}}{936} J(261) + \frac{3}{22} J(241) + \frac{\sqrt{6}}{30} J(221)$$

$$[f_3 f_2] = -\frac{5\sqrt{182}}{572} J(261) + \frac{2\sqrt{15}}{55} J(241) + \frac{\sqrt{10}}{20} J(221)$$

$$[f_2 f_2] = \frac{\sqrt{2730}}{1716} J(261) - \frac{3}{22} J(241) + \frac{\sqrt{6}}{12} J(221)$$

$$[p_0 d_6] p_0 d_6 = \frac{2}{5} J(110) + \frac{6\sqrt{21}}{35} J(130) + \frac{27}{70} J(330)$$

$$[p_0 f_6] = \frac{6\sqrt{5}}{35} J(340) + \frac{27}{70} J(230) + \frac{4\sqrt{105}}{105} J(140) + \frac{3\sqrt{21}}{35} J(120)$$

$$[p_1 p_1] = \frac{\sqrt{5}}{5} J(010) - \frac{3\sqrt{105}}{70} J(030) - \frac{1}{5} J(120) - \frac{3\sqrt{21}}{70} J(230)$$

$$[p_1 d_1] = \frac{\sqrt{3}}{5} J(110) + \frac{3\sqrt{7}}{70} J(130) - \frac{9\sqrt{3}}{70} J(330)$$

$$[p_1 f_1] = -\frac{3\sqrt{30}}{70} J(340) + \frac{9\sqrt{6}}{70} J(230) - \frac{\sqrt{70}}{35} J(140) + \frac{3\sqrt{14}}{35} J(120)$$

$$[d_2 d_2] = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) + \frac{2}{7} J(120) + \frac{6\sqrt{5}}{35} J(140) +$$

$$+ \frac{3\sqrt{21}}{49} J(230) + \frac{9\sqrt{105}}{245} J(340)$$

$$[d_2 f_2] = \frac{5\sqrt{33}}{77} J(350) + \frac{2\sqrt{21}}{35} J(330) + \frac{137}{210} J(130) + \frac{10\sqrt{77}}{231} J(150) + \frac{3\sqrt{21}}{35} J(110)$$

$$[d_1 d_1] = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) + \frac{1}{7} J(120) - \frac{4\sqrt{5}}{35} J(140) +$$

$$+ \frac{3\sqrt{21}}{98} J(230) - \frac{6\sqrt{105}}{245} J(340)$$

$$|d_1 f_1\rangle = -\frac{5\sqrt{66}}{154} J(350) + \frac{\sqrt{42}}{70} J(330) + \frac{34\sqrt{2}}{105} J(130) - \frac{5\sqrt{154}}{231} J(150) + \\ + \frac{2\sqrt{42}}{35} J(110)$$

$$|d_1 d_1\rangle = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) - \frac{2}{7} J(120) + \frac{\sqrt{5}}{35} J(140) - \frac{3\sqrt{21}}{49} J(230) + \\ + \frac{3\sqrt{105}}{490} J(340)$$

$$|d_1 f_2\rangle = \frac{\sqrt{105}}{35} J(110) - \frac{\sqrt{5}}{210} J(130) + \frac{\sqrt{385}}{231} J(150) - \frac{\sqrt{105}}{35} J(330) + \frac{\sqrt{165}}{154} J(350)$$

$$|f_1 f_1\rangle = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) + \frac{4}{15} J(120) + \frac{6\sqrt{5}}{55} J(140) + \frac{20\sqrt{65}}{429} J(160) + \\ + \frac{2\sqrt{21}}{35} J(230) + \frac{9\sqrt{105}}{385} J(340) + \frac{16\sqrt{1365}}{1001} J(360)$$

$$|f_1 f_2\rangle = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) + \frac{1}{5} J(120) + \frac{\sqrt{5}}{55} J(140) - \frac{5\sqrt{65}}{143} J(160) + \\ + \frac{3\sqrt{21}}{70} J(230) + \frac{3\sqrt{105}}{770} J(340) - \frac{15\sqrt{1365}}{2002} J(360)$$

$$|f_2 f_2\rangle = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) - \frac{7\sqrt{5}}{55} J(140) + \frac{2\sqrt{65}}{143} J(160) - \\ - \frac{3\sqrt{105}}{110} J(340) + \frac{3\sqrt{1365}}{1001} J(360)$$

$$|f_2 f_3\rangle = \frac{\sqrt{5}}{5} J(010) + \frac{3\sqrt{105}}{70} J(030) - \frac{1}{3} J(120) + \frac{3\sqrt{5}}{55} J(140) - \frac{\sqrt{65}}{429} J(160) - \\ - \frac{\sqrt{21}}{14} J(230) + \frac{9\sqrt{105}}{770} J(340) - \frac{\sqrt{1365}}{2002} J(360)$$

$$|0_0 d_1\rangle |0_0 d_1\rangle = \frac{3}{10} J(111) + \frac{3\sqrt{14}}{35} J(131) + \frac{12}{35} J(331)$$

$$|0_0 f_1\rangle = \frac{3\sqrt{14}}{35} J(121) + \frac{\sqrt{21}}{14} J(141) + \frac{12}{35} J(231) + \frac{\sqrt{6}}{7} J(341)$$

$$|0_1 d_0\rangle = -\frac{\sqrt{3}}{10} J(111) + \frac{\sqrt{42}}{70} J(131) + \frac{6\sqrt{3}}{35} J(331)$$

$$\begin{aligned}
 |p_1 d_4| &= \frac{3}{10} J (111) + \frac{9\sqrt{14}}{140} J (131) - \frac{3}{35} J (331) \\
 |p_1 f_4| &= -\frac{3\sqrt{21}}{70} J (121) + \frac{\sqrt{14}}{14} J (141) - \frac{3\sqrt{6}}{35} J (231) + \frac{2}{7} J (341) \\
 |p_1 f_4| &= \frac{3\sqrt{35}}{35} J (121) - \frac{\sqrt{210}}{140} J (141) + \frac{3\sqrt{10}}{35} J (231) - \frac{\sqrt{15}}{35} J (341) \\
 |d_4 d_4| &= \frac{\sqrt{3}}{14} J (121) + \frac{3\sqrt{2}}{7} J (141) + \frac{\sqrt{42}}{49} J (231) + \frac{6\sqrt{7}}{49} J (341) \\
 |d_4 f_4| &= \frac{3\sqrt{42}}{70} J (111) + \frac{19\sqrt{3}}{70} J (131) + \frac{\sqrt{2310}}{154} J (151) + \frac{\sqrt{42}}{35} J (331) + \frac{2\sqrt{165}}{77} J (351) \\
 |d_4 d_4| &= \frac{3}{14} J (121) - \frac{\sqrt{6}}{28} J (141) + \frac{3\sqrt{14}}{49} J (231) - \frac{\sqrt{21}}{49} J (341) \\
 |d_4 f_4| &= \frac{3\sqrt{21}}{70} J (111) - \frac{11\sqrt{6}}{210} J (131) + \frac{2\sqrt{1155}}{231} J (151) + \frac{2\sqrt{21}}{105} J (331) + \\
 &\quad + \frac{20\sqrt{330}}{1155} J (351) \\
 |d_1 f_4| &= \frac{3\sqrt{35}}{70} J (111) + \frac{19\sqrt{10}}{140} J (131) - \frac{\sqrt{77}}{77} J (151) + \frac{\sqrt{35}}{35} J (331) - \frac{2\sqrt{22}}{77} J (351) \\
 |d_1 f_4| &= -\frac{3\sqrt{14}}{140} J (111) + \frac{4}{35} J (131) - \frac{\sqrt{770}}{308} J (151) + \frac{2\sqrt{14}}{35} J (331) - \frac{\sqrt{55}}{77} J (351) \\
 |d_2 f_4| &= \frac{3\sqrt{210}}{140} J (111) + \frac{11\sqrt{15}}{210} J (131) + \frac{\sqrt{462}}{924} J (151) - \frac{\sqrt{210}}{105} J (331) + \frac{\sqrt{33}}{231} J (351) \\
 |f_4 f_4| &= \frac{\sqrt{6}}{30} J (121) + \frac{3}{22} J (141) + \frac{5\sqrt{2730}}{936} J (161) + \frac{2\sqrt{21}}{105} J (231) + \\
 &\quad + \frac{3\sqrt{14}}{77} J (341) + \frac{5\sqrt{195}}{234} J (361) \\
 |f_1 f_4| &= \frac{\sqrt{10}}{10} J (121) + \frac{2\sqrt{15}}{55} J (141) - \frac{5\sqrt{182}}{572} J (161) + \frac{\sqrt{35}}{35} J (231) + \\
 &\quad + \frac{4\sqrt{210}}{385} J (341) - \frac{5\sqrt{13}}{113} J (361) \\
 |f_2 f_4| &= \frac{\sqrt{6}}{12} J (121) - \frac{3}{22} J (141) + \frac{\sqrt{2730}}{1716} J (161) + \frac{\sqrt{21}}{21} J (231) - \frac{3\sqrt{14}}{77} J (341) + \\
 &\quad + \frac{\sqrt{195}}{429} J (361)
 \end{aligned}$$

$$(p_0 d_2 | p_0 d_2) = \frac{3}{14} J \quad (332)$$

$$(p_0 f_2) = \frac{3}{14} J \quad (232) + \frac{\sqrt{3}}{7} J \quad (342)$$

$$(p_1 p_1) = \frac{3\sqrt{35}}{70} J \quad (232)$$

$$(p_1 d_1) = \frac{3}{14} J \quad (332)$$

$$(p_1 f_1) = -\frac{3}{14} J \quad (232) + \frac{\sqrt{30}}{28} J \quad (342)$$

$$(p_1 f_2) = \frac{3\sqrt{6}}{28} J \quad (232) - \frac{\sqrt{2}}{28} J \quad (342)$$

$$(d_0 d_2) = -\frac{\sqrt{105}}{49} J \quad (232) + \frac{3\sqrt{35}}{98} J \quad (342)$$

$$(d_0 f_2) = \frac{\sqrt{1155}}{154} J \quad (332)$$

$$(d_1 d_1) = \frac{3\sqrt{35}}{98} J \quad (232) + \frac{\sqrt{105}}{49} J \quad (342)$$

$$(d_1 f_1) = \frac{\sqrt{14}}{28} J \quad (332) + \frac{5\sqrt{154}}{308} J \quad (352)$$

$$(d_1 f_2) = -\frac{\sqrt{21}}{21} J \quad (332) + \frac{5\sqrt{231}}{462} J \quad (352)$$

$$(f_0 f_2) = -\frac{\sqrt{21}}{21} J \quad (232) - \frac{3\sqrt{7}}{154} J \quad (242) + \frac{\sqrt{78}}{39} J \quad (262)$$

$$(f_1 f_1) = \frac{\sqrt{35}}{35} J \quad (232) + \frac{\sqrt{105}}{77} J \quad (342) + \frac{5\sqrt{130}}{286} J \quad (362)$$

$$(f_1 f_2) = -\frac{\sqrt{21}}{42} J \quad (232) + \frac{9\sqrt{7}}{154} J \quad (342) - \frac{5\sqrt{78}}{858} J \quad (362)$$

$$(p_0 f_2 | p_0 f_2) = \frac{27}{70} J \quad (220) + \frac{12\sqrt{5}}{35} J \quad (240) + \frac{8}{21} J \quad (440)$$

$$(p_1 p_1) = \frac{3\sqrt{105}}{70} J \quad (020) - \frac{2\sqrt{21}}{21} J \quad (040) - \frac{3\sqrt{21}}{70} J \quad (220) - \frac{2\sqrt{105}}{105} J \quad (240)$$

$$|p_1 d_1| = \frac{9\sqrt{7}}{70} J(120) + \frac{2\sqrt{35}}{35} J(140) - \frac{9\sqrt{3}}{70} J(230) - \frac{2\sqrt{15}}{35} J(340)$$

$$|p_1 f_1| = \frac{9\sqrt{6}}{70} J(220) + \frac{\sqrt{30}}{70} J(240) - \frac{2\sqrt{6}}{21} J(440)$$

$$|d_0 d_0| = \frac{2\sqrt{21}}{21} J(040) + \frac{3\sqrt{21}}{49} J(220) + \frac{47\sqrt{105}}{735} J(240) - \frac{4\sqrt{21}}{49} J(440) + \frac{3\sqrt{105}}{70} J(020)$$

$$|d_0 f_0| = \frac{27}{70} J(120) + \frac{6\sqrt{5}}{35} J(140) + \frac{2\sqrt{21}}{35} J(230) + \frac{5\sqrt{33}}{77} J(250) + \frac{8\sqrt{105}}{315} J(340) + \frac{20\sqrt{165}}{693} J(450)$$

$$|d_1 d_1| = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) + \frac{3\sqrt{21}}{98} J(220) - \frac{8\sqrt{105}}{735} J(240) - \frac{8\sqrt{21}}{147} J(440)$$

$$|d_1 f_1| = \frac{9\sqrt{2}}{35} J(120) + \frac{4\sqrt{10}}{35} J(140) + \frac{\sqrt{42}}{70} J(230) - \frac{5\sqrt{66}}{154} J(250) + \frac{2\sqrt{210}}{315} J(340) - \frac{10\sqrt{330}}{693} J(450)$$

$$|d_1 d_2| = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) - \frac{3\sqrt{21}}{49} J(220) - \frac{31\sqrt{105}}{1470} J(240) + \frac{2\sqrt{21}}{147} J(440)$$

$$|d_1 f_2| = \frac{9\sqrt{5}}{70} J(120) + \frac{2}{7} J(140) - \frac{\sqrt{105}}{35} J(230) + \frac{\sqrt{165}}{154} J(250) - \frac{4\sqrt{21}}{63} J(340) + \frac{10\sqrt{33}}{693} J(450)$$

$$|f_0 f_0| = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) + \frac{2\sqrt{21}}{35} J(220) + \frac{169\sqrt{105}}{3465} J(240) + \frac{10\sqrt{1365}}{1001} J(260) + \frac{4\sqrt{21}}{77} J(440) + \frac{200\sqrt{273}}{9009} J(460)$$

$$|f_1 f_1\rangle = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) + \frac{3\sqrt{21}}{70} J(220) + \frac{53\sqrt{105}}{2310} J(240) - \\ - \frac{15\sqrt{1365}}{2002} J(260) + \frac{2\sqrt{21}}{231} J(440) - \frac{50\sqrt{273}}{3003} J(460)$$

$$|f_2 f_1\rangle = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) - \frac{3\sqrt{105}}{110} J(240) + \frac{3\sqrt{1365}}{1001} J(260) - \\ - \frac{2\sqrt{21}}{33} J(440) + \frac{20\sqrt{273}}{3003} J(460)$$

$$|f_3 f_1\rangle = \frac{3\sqrt{105}}{70} J(020) + \frac{2\sqrt{21}}{21} J(040) - \frac{\sqrt{21}}{14} J(220) - \frac{139\sqrt{105}}{6939} J(240) - \\ - \frac{\sqrt{1365}}{2002} J(260) + \frac{2\sqrt{21}}{177} J(440) - \frac{10\sqrt{273}}{9009} J(460)$$

$$(p_0 f_1 | p_0 f_1) = \frac{12}{35} J(221) + \frac{2\sqrt{6}}{7} J(241) + \frac{5}{14} J(441)$$

$$|p_1 d_0\rangle = -\frac{\sqrt{42}}{35} J(121) - \frac{\sqrt{7}}{14} J(141) + \frac{6\sqrt{3}}{35} J(231) + \frac{3\sqrt{2}}{14} J(341)$$

$$|p_1 d_1\rangle = \frac{3\sqrt{14}}{35} J(121) + \frac{\sqrt{21}}{14} J(141) - \frac{3}{35} J(231) - \frac{\sqrt{6}}{28} J(341)$$

$$|p_1 f_0\rangle = -\frac{3\sqrt{6}}{35} J(221) + \frac{1}{14} J(241) + \frac{5\sqrt{6}}{42} J(441)$$

$$|p_1 f_1\rangle = \frac{3\sqrt{10}}{35} J(221) + \frac{3\sqrt{15}}{70} J(241) - \frac{\sqrt{10}}{28} J(441)$$

$$|d_0 d_1\rangle = \frac{\sqrt{42}}{49} J(221) + \frac{17\sqrt{7}}{98} J(241) + \frac{5\sqrt{42}}{98} J(441)$$

$$|d_0 f_1\rangle = \frac{6\sqrt{3}}{35} J(121) + \frac{3\sqrt{2}}{14} J(141) + \frac{\sqrt{42}}{35} J(231) + \frac{2\sqrt{165}}{77} J(251) + \\ + \frac{\sqrt{7}}{14} J(341) + \frac{5\sqrt{110}}{154} J(451)$$

$$|d_1 d_1\rangle = \frac{3\sqrt{14}}{49} J(221) - \frac{\sqrt{21}}{49} J(241) + \frac{5\sqrt{21}}{98} J(241) - \frac{5\sqrt{14}}{196} J(441)$$

$$|d_1 f_0\rangle = -\frac{3\sqrt{6}}{35} J(121) - \frac{3}{14} J(141) + \frac{2\sqrt{21}}{105} J(231) + \frac{4\sqrt{330}}{231} J(251) + \\ + \frac{\sqrt{14}}{42} J(341) + \frac{10\sqrt{55}}{231} J(451)$$

$$|d_4 f_4\rangle = \frac{3\sqrt{10}}{35} J(121) + \frac{\sqrt{15}}{14} J(141) + \frac{\sqrt{35}}{35} J(231) - \frac{2\sqrt{22}}{77} J(251) + \\ + \frac{\sqrt{210}}{84} J(341) - \frac{5\sqrt{33}}{231} J(451)$$

$$|d_4 f_4\rangle = -\frac{3}{35} J(121) - \frac{\sqrt{6}}{28} J(141) + \frac{2\sqrt{14}}{35} J(231) - \frac{\sqrt{55}}{77} J(251) + \\ + \frac{\sqrt{21}}{21} J(341) - \frac{5\sqrt{330}}{924} J(451)$$

$$|d_4 f_4\rangle = \frac{3\sqrt{15}}{35} J(121) + \frac{3\sqrt{10}}{28} J(141) - \frac{\sqrt{210}}{105} J(231) - \frac{\sqrt{33}}{231} J(251) - \\ - \frac{\sqrt{35}}{42} J(341) + \frac{5\sqrt{22}}{924} J(451)$$

$$|f_6 f_4\rangle = \frac{2\sqrt{21}}{105} J(221) + \frac{3\sqrt{14}}{77} J(241) + \frac{5\sqrt{195}}{234} J(261) + \frac{\sqrt{14}}{42} J(241) + \\ + \frac{5\sqrt{21}}{154} J(441) + \frac{25\sqrt{130}}{936} J(461)$$

$$|f_4 f_4\rangle = \frac{\sqrt{35}}{35} J(221) + \frac{103\sqrt{210}}{4620} J(241) - \frac{5\sqrt{13}}{143} J(261) + \frac{2\sqrt{35}}{77} J(441) - \\ - \frac{25\sqrt{78}}{1716} J(461)$$

$$|f_4 f_4\rangle = \frac{\sqrt{21}}{21} J(221) + \frac{19\sqrt{14}}{924} J(241) + \frac{\sqrt{195}}{429} J(261) - \frac{5\sqrt{21}}{154} J(441) + \\ + \frac{5\sqrt{130}}{1716} J(461)$$

$$(0_6 f_4 | p_6 f_4\rangle = \frac{3}{14} J(222) + \frac{2\sqrt{3}}{7} J(242) + \frac{2}{7} J(442)$$

$$|p_1 p_1\rangle = \frac{3\sqrt{35}}{70} J(222) + \frac{\sqrt{105}}{35} J(442)$$

$$|p_1 d_4\rangle = \frac{3}{14} J(232) + \frac{\sqrt{3}}{7} J(342)$$

$$|p_1 f_4\rangle = -\frac{3\sqrt{10}}{140} J(222) + \frac{3\sqrt{30}}{140} J(242) + \frac{\sqrt{10}}{14} J(442)$$

$$|p_1 f_3\rangle = \frac{3\sqrt{6}}{28} J(222) + \frac{5\sqrt{2}}{28} J(242) - \frac{\sqrt{6}}{42} J(442)$$

$$|d_4 d_2\rangle = -\frac{\sqrt{15}}{49} J(222) - \frac{\sqrt{35}}{98} J(242) + \frac{\sqrt{105}}{49} J(442)$$

$$|d_4 f_3\rangle = \frac{\sqrt{1155}}{154} J(252) + \frac{\sqrt{385}}{77} J(452)$$

$$|d_4 d_1\rangle = \frac{3\sqrt{35}}{98} J(221) + \frac{2\sqrt{105}}{49} J(242) + \frac{2\sqrt{35}}{49} J(442)$$

$$|d_4 f_1\rangle = \frac{\sqrt{14}}{28} J(232) + \frac{5\sqrt{134}}{308} J(252) + \frac{\sqrt{42}}{42} J(342) + \frac{5\sqrt{462}}{462} J(452)$$

$$|d_4 f_2\rangle = \frac{\sqrt{210}}{84} J(232) - \frac{\sqrt{2310}}{924} J(252) + \frac{\sqrt{70}}{42} J(342) - \frac{\sqrt{770}}{462} J(452)$$

$$|d_4 f_4\rangle = -\frac{\sqrt{21}}{21} J(232) + \frac{3\sqrt{231}}{462} J(252) - \frac{2\sqrt{7}}{21} J(342) + \frac{5\sqrt{77}}{231} J(452)$$

$$|f_4 f_2\rangle = -\frac{\sqrt{21}}{21} J(222) - \frac{53\sqrt{7}}{462} J(242) + \frac{\sqrt{78}}{39} J(262) - \frac{\sqrt{21}}{77} J(442) + \frac{2\sqrt{26}}{39} J(462)$$

$$|f_1 f_1\rangle = \frac{\sqrt{35}}{35} J(222) + \frac{37\sqrt{105}}{1155} J(242) + \frac{5\sqrt{130}}{286} J(262) + \frac{2\sqrt{35}}{77} J(442) + \\ + \frac{5\sqrt{390}}{429} J(462)$$

$$|f_1 f_3\rangle = -\frac{\sqrt{21}}{42} J(222) + \frac{5\sqrt{7}}{462} J(242) - \frac{5\sqrt{78}}{858} J(262) + \frac{3\sqrt{21}}{77} J(442) - \\ - \frac{5\sqrt{26}}{429} J(462)$$

$$|p_1 f_1\rangle |p_1 f_3\rangle = \frac{3}{18} J(443)$$

$$|p_1 d_4\rangle = \frac{\sqrt{42}}{28} J(343)$$

$$|p_1 f_2\rangle = \frac{\sqrt{6}}{12} J(443)$$

$$|d_4 f_3\rangle = -\frac{\sqrt{15}}{18} J(343) + \frac{\sqrt{165}}{99} J(453)$$

$$|d_1 d_2 = \frac{\sqrt{210}}{84} J (443)$$

$$|d_1 f_2 = \frac{\sqrt{30}}{36} J (343) + \frac{\sqrt{330}}{99} J (453)$$

$$|d_2 f_1 = -\frac{\sqrt{3}}{18} J (343) + \frac{5\sqrt{33}}{198} J (453)$$

$$|f_2 f_2 = -\frac{\sqrt{21}}{22} J (443) + \frac{5\sqrt{91}}{429} J (463)$$

$$|f_1 f_2 = \frac{\sqrt{21}}{66} J (443) + \frac{5\sqrt{91}}{286} J (463)$$

$$(P_1 P_1 | P_1 P_1) = \frac{1}{2} J (000) - \frac{\sqrt{5}}{5} J (020) + \frac{1}{10} J (220) + \frac{3}{10} J (222)$$

$$|P_1 d_1 = \frac{\sqrt{15}}{10} J (010) - \frac{3\sqrt{35}}{70} J (030) - \frac{\sqrt{3}}{10} J_1(120) + \frac{3\sqrt{7}}{70} J (230) + \frac{3\sqrt{35}}{70} J (232)$$

$$|P_1 f_1 = \frac{3\sqrt{70}}{70} J (020) - \frac{\sqrt{14}}{14} J (040) - \frac{3\sqrt{14}}{70} J (220) - \frac{3\sqrt{14}}{140} J (222) + \\ + \frac{\sqrt{70}}{70} J (240) + \frac{\sqrt{42}}{28} J (242)$$

$$P_1 f_2 = \frac{3\sqrt{210}}{140} J (222) - \frac{\sqrt{70}}{140} J (242)$$

$$|d_2 d_2 = \frac{1}{2} J (000) + \frac{3\sqrt{5}}{70} J (020) + \frac{3}{7} J (040) - \frac{1}{7} J (220) - \frac{3\sqrt{5}}{35} J (240)$$

$$|d_2 d_2 = -\frac{\sqrt{3}}{7} J (222) + \frac{3}{14} J (242)$$

$$|d_2 f_2 = \frac{3\sqrt{105}}{70} J (010) + \frac{2\sqrt{5}}{15} J (030) + \frac{5\sqrt{385}}{231} J (050) - \frac{3\sqrt{21}}{70} J (120) - \\ - \frac{2}{15} J (230) - \frac{5\sqrt{77}}{231} J (250)$$

$$|d_2 f_2 = \frac{\sqrt{33}}{22} J (252)$$

$$|d_1 d_1 = \frac{1}{2} J (000) - \frac{\sqrt{5}}{35} J (020) - \frac{2}{7} J (040) - \frac{1}{14} J (220) + \frac{3}{14} J (222) + \\ + \frac{2\sqrt{5}}{35} J (240) + \frac{\sqrt{3}}{7} J (242)$$

$$|d_1 f_3\rangle = \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) - \frac{\sqrt{42}}{35} J(120) - \\ - \frac{\sqrt{2}}{30} J(230) + \frac{\sqrt{10}}{20} J(232) + \frac{5\sqrt{154}}{462} J(250) + \frac{\sqrt{110}}{44} J(252)$$

$$|d_1 f_3\rangle = \frac{\sqrt{6}}{12} J(232) - \frac{\sqrt{66}}{132} J(252)$$

$$|d_1 d_2\rangle = \frac{1}{2} J(000) - \frac{17\sqrt{5}}{70} J(020) + \frac{1}{14} J(040) + \frac{1}{7} J(220) - \frac{\sqrt{5}}{70} J(240)$$

$$|d_2 f_6\rangle = -\frac{\sqrt{15}}{15} J(232) + \frac{5\sqrt{165}}{330} J(252)$$

$$|d_2 f_3\rangle = \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) - \frac{\sqrt{105}}{70} J(120) + \\ + \frac{\sqrt{5}}{15} J(230) - \frac{\sqrt{385}}{462} J(250)$$

$$|f_6 f_3\rangle = \frac{1}{2} J(000) + \frac{\sqrt{5}}{30} J(020) + \frac{3}{11} J(040) + \frac{50\sqrt{13}}{462} J(060) - \frac{2}{15} J(220) - \\ - \frac{3\sqrt{5}}{55} J(240) - \frac{10\sqrt{65}}{429} J(260)$$

$$|f_6 f_3\rangle = -\frac{\sqrt{15}}{15} J(222) - \frac{3\sqrt{5}}{110} J(242) + \frac{\sqrt{2730}}{195} J(262)$$

$$|f_1 f_1\rangle = \frac{1}{2} J(000) + \frac{1}{22} J(040) - \frac{25\sqrt{13}}{286} J(060) - \frac{1}{10} J(220) + \frac{1}{5} J(222) - \\ - \frac{\sqrt{5}}{110} J(240) + \frac{\sqrt{3}}{11} J(242) + \frac{5\sqrt{65}}{286} J(260) + \frac{5\sqrt{182}}{286} J(262)$$

$$|f_1 f_3\rangle = -\frac{\sqrt{15}}{30} J(222) + \frac{9\sqrt{5}}{110} J(242) - \frac{\sqrt{2730}}{858} J(262)$$

$$|f_2 f_2\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{10} J(020) - \frac{7}{22} J(040) + \frac{5\sqrt{13}}{143} J(060) + \\ + \frac{7\sqrt{5}}{110} J(240) - \frac{5\sqrt{65}}{715} J(260)$$

$$|f_2 f_3\rangle = \frac{1}{2} J(000) - \frac{4\sqrt{5}}{15} J(020) + \frac{3}{22} J(040) - \frac{5\sqrt{13}}{858} J(060) + \frac{1}{6} J(220) - \\ - \frac{3\sqrt{5}}{110} J(240) + \frac{\sqrt{65}}{858} J(260)$$

$$(p_1 d_0 | p_1 d_0) = \frac{1}{10} J(111) - \frac{3\sqrt{14}}{35} J(131) + \frac{9}{35} J(331)$$

$$|p_1 d_0\rangle = -\frac{\sqrt{3}}{10} J(111) + \frac{\sqrt{42}}{26} J(131) - \frac{3\sqrt{3} J(331)}{70}$$

$$|p_1 f_0\rangle = \frac{3\sqrt{7}}{70} J(121) - \frac{\sqrt{42}}{42} J(141) - \frac{9\sqrt{2}}{70} J(231) + \frac{\sqrt{3}}{7} J(341)$$

$$|p_1 f_0\rangle = -\frac{\sqrt{105}}{70} J(121) + \frac{\sqrt{70}}{140} J(141) + \frac{3\sqrt{30}}{70} J(231) - \frac{3\sqrt{5}}{70} J(341)$$

$$|d_4 d_0\rangle = -\frac{1}{14} J(121) - \frac{\sqrt{6}}{14} J(141) + \frac{3\sqrt{14}}{98} J(231) + \frac{3\sqrt{21}}{49} J(341)$$

$$|d_4 f_0\rangle = -\frac{3\sqrt{14}}{70} J(111) + \frac{11}{70} J(131) - \frac{\sqrt{770}}{154} J(151) + \frac{3\sqrt{14}}{70} J(331) + \frac{3\sqrt{55}}{77} J(351)$$

$$|d_4 d_0\rangle = -\frac{\sqrt{3}}{14} J(121) + \frac{\sqrt{2}}{28} J(141) + \frac{3\sqrt{42}}{98} J(231) - \frac{3\sqrt{7}}{98} J(341)$$

$$|d_4 f_0\rangle = \frac{3\sqrt{7}}{70} J(111) - \frac{17\sqrt{2}}{105} J(131) - \frac{2\sqrt{385}}{231} J(151) + \frac{\sqrt{7}}{35} J(331) + \frac{2\sqrt{110}}{77} J(351)$$

$$|d_4 f_0\rangle = -\frac{\sqrt{105}}{70} J(111) + \frac{11\sqrt{30}}{420} J(131) + \frac{\sqrt{231}}{231} J(151) + \frac{\sqrt{105}}{70} J(331) - \frac{3\sqrt{66}}{231} J(351)$$

$$|d_4 f_0\rangle = \frac{\sqrt{42}}{140} J(111) - \frac{23\sqrt{3}}{210} J(131) + \frac{\sqrt{2310}}{924} J(151) + \frac{\sqrt{42}}{35} J(331) - \frac{\sqrt{165}}{154} J(351)$$

$$|d_4 f_0\rangle = -\frac{3\sqrt{70}}{140} J(111) + \frac{17\sqrt{5}}{105} J(131) - \frac{\sqrt{154}}{924} J(151) - \frac{\sqrt{70}}{70} J(331) + \frac{\sqrt{11}}{154} J(351)$$

$$|f_0 f_0\rangle = -\frac{\sqrt{2}}{30} J(121) - \frac{\sqrt{3}}{22} J(141) - \frac{5\sqrt{910}}{936} J(161) + \frac{3\sqrt{42}}{154} J(341) +$$

$$\frac{5\sqrt{65}}{156} J(361) + \frac{\sqrt{7}}{35} J(231)$$

$$|f_0 f_0\rangle = -\frac{\sqrt{30}}{60} J(121) - \frac{2\sqrt{5}}{55} J(141) + \frac{3\sqrt{546}}{1716} J(161) + \frac{\sqrt{105}}{70} J(231) +$$

$$+ \frac{6\sqrt{70}}{385} J(341) - \frac{5\sqrt{39}}{286} J(361)$$

$$|f_3 f_4) = -\frac{\sqrt{2}}{12} J(121) + \frac{\sqrt{3}}{22} J(141) - \frac{\sqrt{916}}{1716} J(161) + \frac{\sqrt{7}}{14} J(231) - \\ - \frac{3\sqrt{42}}{154} J(341) + \frac{\sqrt{65}}{286} J(361)$$

$$(p_1 d_1 | p_1 d_1) = \frac{3}{10} J(110) - \frac{3\sqrt{21}}{35} J(130) + \frac{9}{70} J(330) + \frac{3}{14} J(332)$$

$$|p_1 f_1) = \frac{3\sqrt{42}}{70} J(120) - \frac{\sqrt{210}}{70} J(140) - \frac{9\sqrt{2}}{70} J(230) - \frac{3\sqrt{10}}{140} J(232) + \\ + \frac{3\sqrt{10}}{70} J(340) + \frac{\sqrt{30}}{28} J(342)$$

$$|p_1 f_2) = \frac{3\sqrt{6}}{28} J(232) - \frac{\sqrt{2}}{14} J(342)$$

$$|d_4 d_4) = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) + \frac{\sqrt{3}}{7} J(120) + \frac{3\sqrt{15}}{35} J(140) - \\ - \frac{3\sqrt{7}}{49} J(230) - \frac{9\sqrt{35}}{245} J(340)$$

$$|d_4 d_2) = -\frac{\sqrt{105}}{49} J(232) + \frac{3\sqrt{35}}{98} J(342)$$

$$|d_4 f_4) = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) + \frac{\sqrt{3}}{7} J(120) + \frac{3\sqrt{15}}{35} J(140) - \\ - \frac{3\sqrt{7}}{49} J(230) - \frac{9\sqrt{35}}{245} J(340)$$

$$|d_4 f_2) = \frac{\sqrt{1155}}{154} J(352)$$

$$|d_1 d_1) = \frac{\sqrt{15}}{10} J(010) + \frac{\sqrt{3}}{14} J(120) - \frac{3\sqrt{35}}{70} J(030) - \frac{2\sqrt{15}}{35} J(140) - \\ - \frac{3\sqrt{7}}{98} J(230) + \frac{6\sqrt{35}}{245} J(340) + \frac{3\sqrt{35}}{98} J(232) + \frac{\sqrt{105}}{49} J(342)$$

$$|d_1 f_1) = \frac{3\sqrt{14}}{35} J(110) + \frac{\sqrt{6}}{30} J(130) - \frac{5\sqrt{462}}{462} J(150) - \frac{\sqrt{14}}{70} J(330) + \\ + \frac{5\sqrt{22}}{154} J(350) + \frac{\sqrt{14}}{28} J(332) + \frac{5\sqrt{154}}{308} J(352)$$

$$|d_1 f_2) = \frac{\sqrt{210}}{84} J(332) - \frac{\sqrt{2310}}{924} J(352)$$

$$|d_4 d_4| = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) - \frac{\sqrt{3}}{7} J(120) + \frac{\sqrt{15}}{70} J(140) + \\ + \frac{5\sqrt{7}}{49} J(230) - \frac{3\sqrt{35}}{14} J(340)$$

$$|d_4 f_4| = -\frac{\sqrt{21}}{21} J(332) + \frac{5\sqrt{231}}{462} J(352)$$

$$|d_4 f_4| = \frac{3\sqrt{35}}{70} J(110) - \frac{23\sqrt{15}}{210} J(130) + \frac{\sqrt{1155}}{462} J(150) + \frac{\sqrt{35}}{35} J(330) - \\ - \frac{\sqrt{55}}{154} J(350)$$

$$|f_4 f_4| = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) + \frac{2\sqrt{3}}{15} J(120) + \frac{3\sqrt{15}}{55} J(140) + \\ + \frac{10\sqrt{195}}{429} J(160) - \frac{2\sqrt{7}}{35} J(230) - \frac{9\sqrt{35}}{385} J(340) - \frac{2\sqrt{455}}{1001} J(360)$$

$$|f_4 f_4| = -\frac{\sqrt{21}}{21} J(232) - \frac{3\sqrt{7}}{154} J(342) + \frac{\sqrt{78}}{39} J(362)$$

$$|f_4 f_4| = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) + \frac{\sqrt{3}}{10} J(120) + \frac{\sqrt{15}}{110} J(140) - \\ - \frac{5\sqrt{195}}{286} J(160) - \frac{3\sqrt{7}}{70} J(230) + \frac{\sqrt{35}}{35} J(232) - \frac{3\sqrt{35}}{770} J(340) + \\ + \frac{\sqrt{105}}{77} J(342) + \frac{15\sqrt{455}}{2602} J(360) + \frac{5\sqrt{130}}{286} J(362)$$

$$|f_4 f_4| = -\frac{\sqrt{21}}{42} J(232) + \frac{9\sqrt{7}}{154} J(342) - \frac{5\sqrt{78}}{858} J(362)$$

$$|f_4 f_4| = \frac{\sqrt{15}}{10} J(010) - \frac{3\sqrt{35}}{70} J(030) - \frac{7\sqrt{15}}{110} J(140) + \frac{\sqrt{195}}{143} J(160) + \\ + \frac{3\sqrt{35}}{110} J(340) - \frac{3\sqrt{455}}{1001} J(360)$$

$$|f_4 f_4| = \frac{\sqrt{15}}{10} J(010) - \frac{\sqrt{3}}{6} J(120) + \frac{3\sqrt{15}}{110} J(140) - \frac{\sqrt{195}}{858} J(160) - \\ - \frac{3\sqrt{35}}{70} J(030) + \frac{\sqrt{7}}{14} J(230) - \frac{9\sqrt{35}}{770} J(340) + \frac{\sqrt{455}}{2602} J(360)$$

$$(p_1 d_3 | p_1 d_3) = \frac{3}{10} J(111) - \frac{3\sqrt{14}}{70} J(131) + \frac{3}{140} J(331) + \frac{9}{28} J(333)$$

$$| p_1 f_3) = -\frac{3\sqrt{21}}{70} J(121) + \frac{\sqrt{14}}{14} J(141) + \frac{3\sqrt{6}}{140} J(231) - \frac{1}{14} J(341)$$

$$| p_1 f_3) = \frac{3\sqrt{35}}{70} J(121) - \frac{\sqrt{210}}{140} J(141) - \frac{3\sqrt{10}}{140} J(231) + \frac{\sqrt{15}}{110} J(341) + \frac{3\sqrt{7}}{28} J(343)$$

$$| d_4 d_4) = \frac{\sqrt{3}}{14} J(121) + \frac{3\sqrt{2}}{14} J(141) - \frac{\sqrt{42}}{196} J(231) - \frac{3\sqrt{7}}{98} J(343)$$

$$| d_4 f_4) = \frac{3\sqrt{42}}{70} J(111) + \frac{\sqrt{3}}{10} J(131) + \frac{\sqrt{2310}}{154} J(151) - \frac{3\sqrt{3}}{70} J(131) - \frac{\sqrt{42}}{140} J(331) - \frac{\sqrt{165}}{154} J(351)$$

$$| d_4 f_4) = \frac{\sqrt{770}}{154} J(353) - \frac{\sqrt{70}}{28} J(333)$$

$$| d_1 d_4) = \frac{3}{14} J(121) - \frac{\sqrt{6}}{28} J(141) - \frac{3\sqrt{14}}{196} J(231) + \frac{\sqrt{21}}{196} J(341) + \frac{3\sqrt{5}}{28} J(343)$$

$$| d_1 f_4) = -\frac{3\sqrt{21}}{70} J(151) + \frac{23\sqrt{6}}{420} J(131) + \frac{2\sqrt{1155}}{231} J(151) - \frac{\sqrt{21}}{210} J(331) - \frac{\sqrt{330}}{231} J(351)$$

$$| d_1 f_4) = \frac{3\sqrt{35}}{70} J(111) + \frac{\sqrt{10}}{35} J(131) - \frac{\sqrt{77}}{77} J(151) - \frac{\sqrt{35}}{140} J(331) + \frac{\sqrt{22}}{154} J(351) + \frac{\sqrt{35}}{28} J(333) + \frac{\sqrt{385}}{77} J(353)$$

$$| d_2 f_4) = -\frac{3\sqrt{14}}{140} J(111) + \frac{31}{140} J(131) - \frac{\sqrt{770}}{308} J(151) - \frac{\sqrt{14}}{70} J(331) + \frac{\sqrt{55}}{308} J(351) - \frac{\sqrt{14}}{28} J(333) + \frac{5\sqrt{154}}{308} J(353)$$

$$| d_2 f_4) = \frac{3\sqrt{210}}{140} J(111) - \frac{23\sqrt{15}}{420} J(131) + \frac{\sqrt{462}}{924} J(151) + \frac{\sqrt{210}}{420} J(331) - \frac{\sqrt{33}}{924} J(351)$$

$$| f_4 f_4) = \frac{\sqrt{6}}{30} J(121) + \frac{3}{22} J(141) + \frac{5\sqrt{2730}}{936} J(161) - \frac{\sqrt{21}}{210} J(231) - \frac{3\sqrt{14}}{308} J(341) - \frac{5\sqrt{195}}{936} J(361)$$

$$|f_0 f_0\rangle = -\frac{9\sqrt{2}}{44} J(343) + \frac{5\sqrt{78}}{286} J(363)$$

$$|f_1 f_2\rangle = \frac{\sqrt{10}}{20} J(121) + \frac{2\sqrt{15}}{55} J(141) - \frac{5\sqrt{182}}{572} J(161) - \frac{\sqrt{38}}{140} J(231) - \\ - \frac{\sqrt{216}}{385} J(341) + \frac{5\sqrt{13}}{572} J(361) + \frac{3\sqrt{2}}{44} J(343) + \frac{15\sqrt{78}}{572} J(363)$$

$$|f_2 f_3\rangle = \frac{\sqrt{6}}{12} J(121) - \frac{3}{22} J(141) + \frac{\sqrt{2730}}{1716} J(161) - \frac{\sqrt{21}}{84} J(231) + \frac{3\sqrt{14}}{308} J(341) - \\ - \frac{\sqrt{195}}{1716} J(361)$$

$$(p_1 f_0 | p_1 f_0\rangle = \frac{9}{70} J(221) - \frac{\sqrt{6}}{7} J(241) + \frac{5}{21} J(441)$$

$$|p_1 f_2\rangle = -\frac{3\sqrt{15}}{70} J(221) + \frac{13\sqrt{16}}{140} J(241) - \frac{\sqrt{15}}{42} J(441)$$

$$|d_0 d_1\rangle = -\frac{3\sqrt{7}}{98} J(221) - \frac{2\sqrt{42}}{147} J(241) + \frac{5\sqrt{7}}{49} J(441)$$

$$|d_0 f_1\rangle = \frac{\sqrt{3}}{7} J(141) - \frac{9\sqrt{2}}{70} J(121) - \frac{2\sqrt{7}}{70} J(231) - \frac{3\sqrt{110}}{154} J(251) + \\ + \frac{\sqrt{42}}{42} J(341) + \frac{5\sqrt{165}}{231} J(451)$$

$$|d_1 d_2\rangle = -\frac{3\sqrt{21}}{98} J(221) + \frac{13\sqrt{14}}{196} J(241) - \frac{5\sqrt{21}}{294} J(441)$$

$$|d_1 f_0\rangle = \frac{9}{70} J(121) - \frac{\sqrt{14}}{70} J(231) - \frac{2\sqrt{55}}{77} J(251) - \frac{\sqrt{6}}{14} J(141) + \frac{\sqrt{21}}{63} J(341) + \\ + \frac{10\sqrt{330}}{693} J(451)$$

$$|d_1 f_2\rangle = \frac{\sqrt{10}}{14} J(141) + \frac{\sqrt{35}}{42} J(341) - \frac{5\sqrt{22}}{231} J(451)$$

$$|d_2 f_1\rangle = \frac{3\sqrt{6}}{140} J(121) - \frac{\sqrt{21}}{35} J(231) + \frac{\sqrt{330}}{308} J(251) - \frac{1}{14} J(141) + \frac{\sqrt{14}}{21} J(341) - \\ - \frac{5\sqrt{55}}{462} J(451)$$

$$|d_4 f_4| = -\frac{9\sqrt{10}}{140} J(121) + \frac{\sqrt{35}}{70} J(231) - \frac{\sqrt{22}}{308} J(251) + \frac{\sqrt{15}}{14} J(141) - \\ - \frac{\sqrt{210}}{126} J(341) + \frac{5\sqrt{33}}{1386} J(451)$$

$$|f_4 f_4| = -\frac{5\sqrt{21}}{1386} J(241) - \frac{\sqrt{14}}{70} J(221) - \frac{5\sqrt{130}}{312} J(261) + \frac{5\sqrt{14}}{134} J(441) + \\ + \frac{25\sqrt{195}}{1404} J(461)$$

$$|f_4 f_4| = -\frac{\sqrt{210}}{140} J(221) - \frac{6\sqrt{35}}{385} J(241) + \frac{5\sqrt{78}}{572} J(261) + \frac{\sqrt{35}}{42} J(241) + \\ + \frac{2\sqrt{210}}{231} J(441) - \frac{25\sqrt{13}}{858} J(461)$$

$$|f_4 f_4| = -\frac{\sqrt{14}}{28} J(221) + \frac{41\sqrt{21}}{693} J(241) - \frac{\sqrt{130}}{572} J(261) - \frac{5\sqrt{14}}{154} J(441) + \\ + \frac{5\sqrt{195}}{2574} J(461)$$

$$|p_1 f_1| p_1 f_1 = \frac{9}{35} J(220) - \frac{6\sqrt{5}}{35} J(240) + \frac{1}{7} J(440) + \frac{3}{140} J(222) - \frac{\sqrt{3}}{14} J(242) + \\ + \frac{5}{28} J(442)$$

$$|p_1 f_4| = -\frac{3\sqrt{15}}{140} J(222) + \frac{4\sqrt{5}}{35} J(242) - \frac{\sqrt{15}}{84} J(442)$$

$$|d_4 d_4| = \frac{3\sqrt{70}}{70} J(020) + \frac{3\sqrt{14}}{49} J(220) + \frac{4\sqrt{70}}{245} J(240) - \frac{\sqrt{14}}{14} J(040) - \\ - \frac{3\sqrt{14}}{49} J(440)$$

$$|d_4 d_4| = \frac{\sqrt{42}}{98} J(222) - \frac{13\sqrt{14}}{196} J(242) + \frac{5\sqrt{42}}{196} J(442)$$

$$|d_4 f_4| = \frac{9\sqrt{6}}{70} J(120) + \frac{2\sqrt{14}}{35} J(230) + \frac{5\sqrt{22}}{77} J(250) - \frac{3\sqrt{30}}{70} J(140) - \\ - \frac{2\sqrt{70}}{105} J(340) - \frac{5\sqrt{110}}{231} J(450)$$

$$\begin{aligned}
 |d_0 f_2| &= -\frac{\sqrt{462}}{308} J(252) + \frac{5\sqrt{154}}{308} J(452) \\
 |d_1 d_1| &= \frac{3\sqrt{70}}{70} J(020) + \frac{3\sqrt{14}}{98} J(220) - \frac{6\sqrt{70}}{245} J(240) - \frac{\sqrt{14}}{14} J(040) - \\
 &\quad - \frac{\sqrt{70}}{98} J(240) + \frac{2\sqrt{14}}{49} J(440) - \frac{3\sqrt{14}}{196} J(222) + \frac{3\sqrt{42}}{196} J(242) + \frac{5\sqrt{14}}{98} J(442) \\
 |d_1 f_1| &= \frac{6\sqrt{3}}{35} J(120) + \frac{\sqrt{7}}{35} J(230) - \frac{5\sqrt{11}}{77} J(250) - \frac{2\sqrt{15}}{35} J(140) - \\
 &\quad - \frac{\sqrt{35}}{165} J(340) + \frac{5\sqrt{55}}{231} J(450) - \frac{\sqrt{35}}{140} J(232) - \frac{\sqrt{385}}{308} J(252) + \\
 &\quad + \frac{\sqrt{165}}{84} J(342) + \frac{5\sqrt{1155}}{924} J(452) \\
 |d_1 f_2| &= -\frac{\sqrt{21}}{84} J(232) + \frac{\sqrt{231}}{924} J(252) + \frac{5\sqrt{7}}{84} J(342) - \frac{5\sqrt{77}}{924} J(452) \\
 |d_2 d_2| &= \frac{3\sqrt{70}}{70} J(020) - \frac{3\sqrt{14}}{49} J(220) + \frac{3\sqrt{70}}{490} J(240) - \frac{\sqrt{14}}{14} J(040) + \\
 &\quad + \frac{\sqrt{70}}{49} J(240) - \frac{\sqrt{14}}{98} J(440) \\
 |d_2 f_0| &= \frac{\sqrt{210}}{210} J(232) - \frac{\sqrt{2310}}{924} J(252) - \frac{\sqrt{70}}{42} J(342) + \frac{5\sqrt{770}}{924} J(452) \\
 |d_2 f_2| &= \frac{3\sqrt{30}}{70} J(120) - \frac{\sqrt{70}}{35} J(230) + \frac{\sqrt{110}}{154} J(250) - \frac{\sqrt{6}}{14} J(140) + \\
 &\quad + \frac{\sqrt{14}}{21} J(340) - \frac{5\sqrt{22}}{462} J(450) \\
 |f_0 f_0| &= \frac{3\sqrt{70}}{70} J(020) + \frac{2\sqrt{14}}{35} J(220) + \frac{\sqrt{70}}{231} J(240) + \frac{10\sqrt{910}}{1001} J(260) - \\
 &\quad - \frac{\sqrt{14}}{14} J(040) - \frac{3\sqrt{14}}{77} J(440) - \frac{50\sqrt{182}}{3003} J(460) \\
 |f_0 f_2| &= \frac{\sqrt{210}}{420} J(222) - \frac{101\sqrt{70}}{4620} J(242) - \frac{\sqrt{195}}{195} J(262) - \frac{\sqrt{210}}{308} J(442) + \\
 &\quad + \frac{\sqrt{65}}{39} J(462)
 \end{aligned}$$

$$\begin{aligned}
 |f_1 f_3| &= \frac{3\sqrt{70}}{70} J(020) + \frac{3\sqrt{14}}{70} J(220) - \frac{4\sqrt{70}}{385} J(240) - \frac{15\sqrt{910}}{2002} J(260) - \\
 &- \frac{\sqrt{14}}{14} J(040) - \frac{\sqrt{14}}{154} J(440) + \frac{25\sqrt{182}}{2002} J(460) - \frac{\sqrt{14}}{70} J(222) + \\
 &+ \frac{4\sqrt{42}}{231} J(242) - \frac{5\sqrt{13}}{286} J(262) + \frac{5\sqrt{14}}{154} J(442) + \frac{25\sqrt{39}}{858} J(462) \\
 |f_1 f_4| &= -\frac{5\sqrt{65}}{858} J(462) + \frac{3\sqrt{210}}{308} J(442) - \frac{\sqrt{70}}{84} J(242) + \frac{\sqrt{195}}{858} J(262) - \\
 &- \frac{41\sqrt{70}}{2310} J(242) - \frac{\sqrt{210}}{420} J(222)
 \end{aligned}$$

$$\begin{aligned}
 |f_2 f_3| &= \frac{3\sqrt{70}}{70} J(020) - \frac{3\sqrt{70}}{110} J(240) + \frac{3\sqrt{910}}{1001} J(260) - \frac{\sqrt{14}}{14} J(040) + \\
 &+ \frac{\sqrt{14}}{22} J(440) - \frac{5\sqrt{182}}{1001} J(460)
 \end{aligned}$$

$$\begin{aligned}
 |f_2 f_4| &= \frac{3\sqrt{70}}{70} J(020) - \frac{\sqrt{14}}{14} J(040) - \frac{\sqrt{14}}{14} J(220) + \frac{41\sqrt{70}}{1155} J(240) - \\
 &- \frac{\sqrt{910}}{2002} J(260) - \frac{3\sqrt{14}}{154} J(440) + \frac{5\sqrt{182}}{6006} J(460)
 \end{aligned}$$

$$|p_1 f_2| p_1 f_4 = -\frac{\sqrt{6}}{14} J(441) + \frac{3}{14} J(221) + \frac{1}{28} J(441) + \frac{1}{4} J(443)$$

$$|d_6 d_7| = \frac{\sqrt{105}}{98} J(221) + \frac{5\sqrt{70}}{196} J(241) - \frac{\sqrt{105}}{98} J(441)$$

$$\begin{aligned}
 |d_6 f_7| &= \frac{3\sqrt{30}}{70} J(121) - \frac{3\sqrt{5}}{70} J(141) + \frac{\sqrt{105}}{70} J(231) + \frac{5\sqrt{66}}{154} J(251) - \\
 &- \frac{\sqrt{70}}{140} J(341) - \frac{5\sqrt{11}}{154} J(451)
 \end{aligned}$$

$$|d_6 f_8| = -\frac{\sqrt{10}}{12} J(343) + \frac{\sqrt{110}}{66} J(453)$$

$$|d_1 d_2| = \frac{3\sqrt{35}}{98} J(221) - \frac{\sqrt{210}}{98} J(241) + \frac{\sqrt{35}}{196} J(441) + \frac{\sqrt{35}}{28} J(443)$$

$$\begin{aligned}
 |d_1 f_8| &= -\frac{3\sqrt{15}}{70} J(121) + \frac{\sqrt{210}}{210} J(231) + \frac{10\sqrt{33}}{231} J(251) + \frac{3\sqrt{10}}{140} J(141) - \\
 &- \frac{\sqrt{35}}{210} J(341) - \frac{5\sqrt{22}}{231} J(451)
 \end{aligned}$$

$$|d_4 f_0\rangle = \frac{3}{14} J(121) + \frac{\sqrt{14}}{28} J(231) - \frac{\sqrt{55}}{77} J(251) - \frac{\sqrt{6}}{28} J(141) - \\ - \frac{\sqrt{21}}{84} J(341) + \frac{\sqrt{330}}{462} J(451) + \frac{\sqrt{5}}{12} J(343) + \frac{\sqrt{55}}{33} J(453)$$

$$|d_4 f_1\rangle = -\frac{3\sqrt{10}}{140} J(121) + \frac{\sqrt{35}}{35} J(231) - \frac{5\sqrt{22}}{308} J(251) + \frac{\sqrt{15}}{140} J(141) - \\ - \frac{\sqrt{210}}{210} J(341) + \frac{5\sqrt{33}}{924} J(451) - \frac{\sqrt{2}}{12} J(343) + \frac{5\sqrt{22}}{132} J(453)$$

$$|d_4 f_2\rangle = \frac{3\sqrt{6}}{28} J(121) - \frac{\sqrt{21}}{42} J(231) + \frac{\sqrt{330}}{924} J(251) - \frac{3}{28} J(141) + \\ + \frac{\sqrt{14}}{84} J(341) - \frac{\sqrt{55}}{924} J(451)$$

$$|f_4 f_1\rangle = \frac{\sqrt{210}}{210} J(221) + \frac{17\sqrt{35}}{1155} J(241) + \frac{25\sqrt{78}}{936} J(261) - \\ - \frac{\sqrt{210}}{308} J(441) - \frac{25\sqrt{13}}{72} J(461)$$

$$|f_4 f_2\rangle = -\frac{3\sqrt{14}}{44} J(443) + \frac{5\sqrt{546}}{858} J(463)$$

$$|f_1 f_2\rangle = \frac{\sqrt{14}}{28} J(221) + \frac{13\sqrt{21}}{924} J(241) - \frac{5\sqrt{130}}{616} J(261) - \frac{\sqrt{14}}{77} J(441) + \\ + \frac{5\sqrt{195}}{1716} J(461) + \frac{\sqrt{14}}{44} J(443) + \frac{5\sqrt{546}}{572} J(463)$$

$$|f_2 f_2\rangle = \frac{\sqrt{210}}{84} J(221) - \frac{29\sqrt{35}}{924} J(241) + \frac{5\sqrt{78}}{1716} J(261) + \frac{\sqrt{210}}{308} J(441) - \\ - \frac{5\sqrt{13}}{1716} J(461)$$

$$|p_1 f_1\rangle |p_1 f_2\rangle = -\frac{\sqrt{3}}{14} J(242) + \frac{9}{28} J(222) + \frac{1}{84} J(442) + \frac{1}{3} J(444)$$

$$|d_4 d_2\rangle = -\frac{3\sqrt{70}}{98} J(222) + \frac{11\sqrt{210}}{588} J(242) - \frac{\sqrt{70}}{196} J(442)$$

$$|d_4 f_1\rangle = \frac{3\sqrt{770}}{308} J(252) - \frac{\sqrt{2310}}{924} J(452)$$

$$|d_1 d_1\rangle = \frac{3\sqrt{210}}{196} J(222) + \frac{5\sqrt{70}}{196} J(242) - \frac{\sqrt{210}}{294} J(442)$$

$$|d_1 f_1| = \frac{\sqrt{21}}{28} J (232) + \frac{5\sqrt{231}}{308} J (252) - \frac{\sqrt{7}}{84} J (342) - \frac{5\sqrt{77}}{924} J (452)$$

$$|d_1 f_2| = \frac{\sqrt{35}}{28} J (232) - \frac{\sqrt{385}}{308} J (252) - \frac{\sqrt{105}}{252} J (342) + \frac{\sqrt{1155}}{2772} J (452) + \frac{\sqrt{55}}{33} J (454)$$

$$|d_2 d_3| = \frac{\sqrt{210}}{42} J (444)$$

$$|d_2 f_3| = -\frac{\sqrt{14}}{14} J (232) + \frac{5\sqrt{154}}{308} J (252) + \frac{\sqrt{42}}{126} J (342) - \frac{5\sqrt{462}}{2772} J (452)$$

$$|d_2 f_4| = \frac{\sqrt{55}}{33} J (454)$$

$$|f_6 f_3| = -\frac{\sqrt{14}}{14} J (232) - \frac{5\sqrt{42}}{2772} J (242) + \frac{\sqrt{13}}{13} J (262) + \frac{\sqrt{14}}{308} J (442) - \frac{\sqrt{39}}{117} J (462)$$

$$|f_1 f_4| = \frac{\sqrt{210}}{70} J (222) + \frac{17\sqrt{70}}{1155} J (242) + \frac{5\sqrt{195}}{286} J (262) - \frac{\sqrt{210}}{462} J (442) -$$

$$-\frac{5}{858} \sqrt{65} J (462)$$

$$|f_1 f_5| = -\frac{\sqrt{14}}{28} J (232) + \frac{23\sqrt{42}}{693} J (242) - \frac{5\sqrt{13}}{286} J (262) - \frac{3\sqrt{14}}{308} J (442) +$$

$$+\frac{5\sqrt{39}}{2574} J (462) - \frac{\sqrt{14}}{22} J (444) + \frac{5\sqrt{910}}{858} J (464)$$

$$|f_2 f_4| = \frac{\sqrt{210}}{66} J (444) + \frac{5\sqrt{546}}{429} J (464)$$

$$(d_3 d_4 |d_5 d_6) = \frac{2\sqrt{5}}{7} J (020) + \frac{6}{7} J (040) + \frac{12\sqrt{5}}{49} J (240) + \frac{1}{2} J (000) + \frac{10}{49} J (220) +$$

$$+\frac{18}{49} J (440)$$

$$|d_6 f_6| = \frac{3\sqrt{105}}{70} J (010) + \frac{2\sqrt{5}}{15} J (030) + \frac{5\sqrt{385}}{231} J (050) + \frac{3\sqrt{21}}{49} J (120) +$$

$$+\frac{4}{21} J (230) + \frac{50\sqrt{77}}{1617} J (250) + \frac{9\sqrt{105}}{245} J (140) + \frac{4\sqrt{5}}{35} J (340) + \frac{10\sqrt{385}}{539} J (450)$$

$$|d_1 d_1| = \frac{1}{2} J (000) + \frac{3\sqrt{5}}{14} J (020) - \frac{1}{7} J (040) + \frac{5}{49} J (220) - \frac{\sqrt{5}}{49} J (240) - \frac{12}{49} J (440)$$

$$|d_4 f_1| = \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) + \frac{2\sqrt{42}}{49} J(120) + \\ + \frac{\sqrt{2}}{21} J(230) - \frac{25\sqrt{154}}{1617} J(250) + \frac{6\sqrt{210}}{245} J(140) + \frac{\sqrt{10}}{35} J(340) - \frac{5\sqrt{770}}{539} J(450)$$

$$|d_4 d_3| = \frac{1}{2} J(000) + \frac{1}{2} J(010) - \frac{10}{49} J(220) - \frac{5\sqrt{5}}{49} J(240) + \frac{3}{49} J(440)$$

$$|d_4 f_2| = \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) + \frac{\sqrt{150}}{49} J(120) - \frac{2\sqrt{5}}{21} J(230) + \\ + \frac{5\sqrt{385}}{1617} J(250) + \frac{3\sqrt{21}}{49} J(140) - \frac{2}{7} J(340) + \frac{5\sqrt{77}}{539} J(450)$$

$$|f_4 f_4| = \frac{1}{2} J(000) + \frac{29\sqrt{5}}{105} J(020) + \frac{54}{77} J(040) + \frac{50\sqrt{13}}{429} J(060) + \frac{4}{21} J(220) + \\ + \frac{74\sqrt{5}}{385} J(240) + \frac{100\sqrt{65}}{3003} J(260) + \frac{18}{77} J(440) + \frac{100\sqrt{13}}{1001} J(460)$$

$$|f_4 f_3| = \frac{1}{2} J(000) + \frac{17\sqrt{5}}{70} J(020) + \frac{73}{154} J(040) - \frac{25\sqrt{13}}{286} J(060) + \frac{1}{7} J(220) - \\ - \frac{25\sqrt{65}}{1601} J(260) + \frac{38\sqrt{5}}{385} J(240) + \frac{3}{77} J(440) - \frac{75\sqrt{13}}{1001} J(460)$$

$$|f_4 f_2| = \frac{1}{2} J(000) + \frac{\sqrt{5}}{7} J(020) + \frac{17}{154} J(040) + \frac{5\sqrt{13}}{143} J(060) - \frac{\sqrt{5}}{11} J(240) + \\ + \frac{10\sqrt{65}}{1001} J(260) - \frac{3}{11} J(440) + \frac{30\sqrt{13}}{1001} J(460)$$

$$|f_4 f_1| = \frac{1}{2} J(000) - \frac{\sqrt{5}}{42} J(020) + \frac{87}{154} J(040) - \frac{5\sqrt{13}}{858} J(060) - \frac{5}{21} J(220) - \\ - \frac{8\sqrt{5}}{77} J(240) - \frac{5\sqrt{65}}{3003} J(260) + \frac{9}{77} J(440) - \frac{5\sqrt{13}}{1001} J(460)$$

$$(d_4 d_3 | d_4 d_4) = \frac{5}{98} J(221) + \frac{5\sqrt{6}}{49} J(241) + \frac{15}{49} J(441)$$

$$|d_4 f_1| = \frac{3\sqrt{14}}{98} J(121) + \frac{3\sqrt{21}}{49} J(141) + \frac{1}{14} J(231) + \frac{5\sqrt{770}}{1078} J(251) + \\ + \frac{\sqrt{6}}{14} J(341) + \frac{5\sqrt{1155}}{539} J(451)$$

$$|d_1 d_3| = \frac{5\sqrt{3}}{98} J(221) + \frac{25\sqrt{2}}{196} J(241) - \frac{5\sqrt{3}}{98} J(441)$$

$$|d_1 f_3| = -\frac{3\sqrt{7}}{98} J(121) - \frac{3\sqrt{42}}{98} J(141) + \frac{\sqrt{2}}{42} J(231) + \frac{10\sqrt{385}}{1617} J(251) + \\ + \frac{\sqrt{3}}{21} J(341) + \frac{10\sqrt{2310}}{1617} J(451)$$

$$|d_1 f_4| = \frac{\sqrt{105}}{98} J(121) + \frac{3\sqrt{70}}{98} J(141) + \frac{\sqrt{30}}{84} J(231) - \frac{5\sqrt{231}}{1617} J(251) + \\ + \frac{\sqrt{5}}{14} J(341) - \frac{5\sqrt{154}}{539} J(451)$$

$$|d_2 f_4| = -\frac{\sqrt{42}}{196} J(121) - \frac{3\sqrt{7}}{98} J(141) + \frac{\sqrt{5}}{21} J(231) - \frac{5\sqrt{2310}}{6468} J(251) + \\ + \frac{\sqrt{2}}{7} J(341) - \frac{5\sqrt{385}}{1078} J(451)$$

$$|d_2 f_5| = \frac{3\sqrt{70}}{196} J(121) + \frac{3\sqrt{105}}{98} J(141) - \frac{\sqrt{5}}{42} J(231) + \frac{5\sqrt{154}}{6468} J(251) - \\ - \frac{\sqrt{30}}{42} J(341) + \frac{5\sqrt{21}}{3234} J(451)$$

$$|f_3 f_4| = \frac{\sqrt{2}}{42} J(221) + \frac{37\sqrt{3}}{462} J(241) + \frac{25\sqrt{910}}{6552} J(261) + \frac{15\sqrt{2}}{154} J(441) + \\ + \frac{25\sqrt{1365}}{3276} J(461)$$

$$|f_4 f_5| = \frac{\sqrt{30}}{84} J(221) + \frac{15\sqrt{5}}{14} J(241) - \frac{25\sqrt{546}}{12012} J(261) + \frac{2\sqrt{30}}{77} J(441) - \\ - \frac{25\sqrt{91}}{2002} J(461)$$

$$|f_5 f_6| = \frac{5\sqrt{2}}{84} J(221) + \frac{20\sqrt{3}}{231} J(241) + \frac{5\sqrt{910}}{12012} J(261) - \frac{15\sqrt{2}}{154} J(441) + \\ + \frac{5\sqrt{1365}}{6006} J(461)$$

$$|d_4 d_5| = \frac{10}{49} J(222) - \frac{10\sqrt{3}}{49} J(242) + \frac{15}{98} J(442)$$

$$\begin{aligned}
 |d_0 f_0| &= -\frac{5\sqrt{11}}{77} J(252) + \frac{5\sqrt{33}}{154} J(452) \\
 |d_1 d_1| &= -\frac{5\sqrt{3}}{49} J(222) - \frac{5}{98} J(242) - \frac{5\sqrt{3}}{49} J(442) \\
 |d_1 f_1| &= -\frac{\sqrt{30}}{42} J(232) - \frac{5\sqrt{330}}{462} J(252) + \frac{\sqrt{10}}{28} J(342) + \frac{5\sqrt{110}}{308} J(452) \\
 |d_1 f_0| &= -\frac{5\sqrt{2}}{42} J(232) + \frac{5\sqrt{22}}{462} J(252) + \frac{5\sqrt{6}}{84} J(342) - \frac{5\sqrt{66}}{924} J(452) \\
 |d_2 f_0| &= \frac{2\sqrt{5}}{21} J(232) - \frac{5\sqrt{55}}{231} J(252) - \frac{\sqrt{15}}{21} J(342) + \frac{5\sqrt{165}}{462} J(452) \\
 |f_0 f_0| &= \frac{2\sqrt{5}}{21} J(222) - \frac{8\sqrt{15}}{231} J(242) - \frac{2\sqrt{910}}{273} J(262) - \frac{3\sqrt{5}}{154} J(442) + \\
 &\quad + \frac{\sqrt{2730}}{273} J(462) \\
 |f_1 f_0| &= -\frac{2\sqrt{3}}{21} J(222) + \frac{1}{77} J(242) - \frac{25\sqrt{546}}{3003} J(262) + \frac{5\sqrt{3}}{77} J(442) + \\
 &\quad + \frac{25\sqrt{182}}{2002} J(462) \\
 |f_1 f_1| &= \frac{\sqrt{5}}{21} J(222) + \frac{29\sqrt{15}}{462} J(242) + \frac{5\sqrt{910}}{3003} J(262) + \frac{9\sqrt{5}}{154} J(442) - \\
 &\quad - \frac{5\sqrt{2730}}{6006} J(462) \\
 (d_4 f_4) |d_0 f_0| &= \frac{4\sqrt{21}}{35} J(130) + \frac{10\sqrt{33}}{77} J(150) + \frac{27}{70} J(110) + \frac{8}{45} J(330) + \\
 &\quad + \frac{40\sqrt{77}}{693} J(350) + \frac{250}{693} J(550) \\
 |d_1 d_4| &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) + \frac{3\sqrt{21}}{98} J(120) - \\
 &\quad - \frac{6\sqrt{105}}{245} J(140) + \frac{2}{21} J(230) + \frac{25\sqrt{77}}{1617} J(250) - \frac{8\sqrt{5}}{105} J(310) - \frac{20\sqrt{385}}{1617} J(450) \\
 |d_1 f_4| &= \frac{9\sqrt{2}}{35} J(110) + \frac{11\sqrt{42}}{210} J(130) - \frac{5\sqrt{66}}{462} J(150) + \frac{2\sqrt{2}}{45} J(330) - \\
 &\quad - \frac{5\sqrt{154}}{693} J(350) - \frac{125\sqrt{2}}{693} J(550)
 \end{aligned}$$

$$\begin{aligned}
 [d_4 d_4] &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) - \frac{3\sqrt{21}}{49} J(120) + \\
 &+ \frac{3\sqrt{105}}{490} J(140) - \frac{4}{21} J(230) - \frac{50\sqrt{77}}{1617} J(250) + \frac{2\sqrt{5}}{165} J(340) + \frac{5\sqrt{385}}{1617} J(450)
 \end{aligned}$$

$$\begin{aligned}
 [d_4 f_4] &= \frac{9\sqrt{5}}{70} J(110) - \frac{\sqrt{105}}{105} J(130) + \frac{13\sqrt{165}}{462} J(150) - \frac{4\sqrt{5}}{45} J(330) - \\
 &- \frac{8\sqrt{385}}{693} J(350) + \frac{25\sqrt{5}}{693} J(550)
 \end{aligned}$$

$$\begin{aligned}
 [f_4 f_4] &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) + \frac{2\sqrt{21}}{35} J(120) + \\
 &+ \frac{9\sqrt{105}}{385} J(140) + \frac{50\sqrt{1365}}{5005} J(160) + \frac{8}{45} J(230) + \frac{20\sqrt{77}}{693} J(250) + \\
 &+ \frac{4\sqrt{5}}{55} J(340) + \frac{40\sqrt{65}}{1287} J(360) + \frac{10\sqrt{385}}{847} J(450) + \frac{500\sqrt{5005}}{99099} J(560)
 \end{aligned}$$

$$\begin{aligned}
 [f_4 f_4] &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) + \frac{3\sqrt{21}}{70} J(120) + \\
 &+ \frac{3\sqrt{105}}{770} J(140) - \frac{15\sqrt{1365}}{2002} J(160) + \frac{2}{15} J(230) + \frac{5\sqrt{77}}{231} J(250) + \\
 &+ \frac{2\sqrt{5}}{165} J(340) - \frac{10\sqrt{65}}{429} J(360) + \frac{5\sqrt{385}}{2541} J(450) - \frac{125\sqrt{5005}}{33033} J(560)
 \end{aligned}$$

$$\begin{aligned}
 [f_4 f_4] &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) - \frac{5\sqrt{105}}{110} J(140) + \\
 &+ \frac{3\sqrt{1365}}{1001} J(160) - \frac{14\sqrt{5}}{165} J(340) + \frac{4\sqrt{65}}{429} J(360) - \frac{5\sqrt{385}}{363} J(450) + \\
 &+ \frac{50\sqrt{5005}}{33033} J(560)
 \end{aligned}$$

$$\begin{aligned}
 [f_4 f_4] &= \frac{3\sqrt{105}}{70} J(010) + \frac{2\sqrt{5}}{15} J(030) + \frac{5\sqrt{385}}{231} J(050) - \frac{\sqrt{21}}{14} J(120) + \\
 &+ \frac{9\sqrt{105}}{770} J(140) - \frac{\sqrt{1365}}{2002} J(160) - \frac{2}{9} J(230) - \frac{25\sqrt{77}}{693} J(250) + \\
 &+ \frac{2\sqrt{5}}{55} J(340) - \frac{2\sqrt{65}}{1287} J(360) + \frac{5\sqrt{385}}{847} J(450) - \frac{25\sqrt{5005}}{99099} J(560)
 \end{aligned}$$

$$\begin{aligned}
 (d_4 f_1 | d_4 f_1) &= \frac{9}{35} J(111) + \frac{3\sqrt{14}}{35} J(131) + \frac{6\sqrt{55}}{77} J(151) + \frac{1}{10} J(331) + \\
 &\quad + \frac{\sqrt{770}}{77} J(351) + \frac{25}{77} J(551) \\
 |d_4 d_4) &= \frac{3\sqrt{42}}{98} J(121) - \frac{3\sqrt{7}}{98} J(141) + \frac{\sqrt{3}}{14} J(231) + \frac{5\sqrt{2310}}{1078} J(251) - \\
 &\quad - \frac{\sqrt{2}}{28} J(341) - \frac{5\sqrt{385}}{1078} J(451) \\
 |d_4 f_4) &= -\frac{9\sqrt{2}}{70} J(111) - \frac{\sqrt{7}}{70} J(131) + \frac{\sqrt{110}}{154} J(151) + \frac{\sqrt{2}}{30} J(331) + \\
 &\quad + \frac{3\sqrt{385}}{231} J(351) + \frac{50\sqrt{2}}{231} J(551) \\
 |d_4 f_4) &= \frac{3\sqrt{30}}{70} J(111) + \frac{\sqrt{105}}{35} J(131) + \frac{3\sqrt{66}}{154} J(151) + \frac{\sqrt{30}}{60} J(331) + \\
 &\quad + \frac{\sqrt{231}}{154} J(351) - \frac{5\sqrt{30}}{231} J(551) \\
 |d_4 f_4) &= -\frac{3\sqrt{3}}{70} J(111) + \frac{3\sqrt{42}}{140} J(131) - \frac{\sqrt{165}}{77} J(151) + \frac{\sqrt{3}}{15} J(331) + \\
 &\quad + \frac{\sqrt{2310}}{308} J(351) - \frac{25\sqrt{3}}{462} J(551) \\
 |d_4 f_4) &= \frac{9\sqrt{5}}{70} J(111) + \frac{\sqrt{70}}{140} J(131) + \frac{3\sqrt{11}}{77} J(151) - \frac{\sqrt{5}}{30} J(331) - \\
 &\quad - \frac{3\sqrt{154}}{308} J(351) + \frac{5\sqrt{5}}{462} J(551) \\
 |f_4 f_1) &= \frac{\sqrt{7}}{35} J(121) + \frac{3\sqrt{42}}{154} J(141) + \frac{5\sqrt{65}}{156} J(161) + \frac{\sqrt{2}}{30} J(231) + \\
 &\quad + \frac{\sqrt{385}}{231} J(251) + \frac{\sqrt{3}}{22} J(341) + \frac{5\sqrt{910}}{936} J(361) + \frac{5\sqrt{2310}}{1694} J(451) + \frac{125\sqrt{143}}{5148} J(561) \\
 |f_4 f_2) &= \frac{\sqrt{105}}{140} J(121) + \frac{6\sqrt{70}}{385} J(141) - \frac{5\sqrt{39}}{286} J(161) + \frac{\sqrt{30}}{60} J(231) + \\
 &\quad + \frac{5\sqrt{231}}{462} J(251) + \frac{2\sqrt{5}}{35} J(341) - \frac{5\sqrt{546}}{1716} J(361) + \frac{10\sqrt{154}}{847} J(451) - \frac{25\sqrt{2145}}{9438} J(561)
 \end{aligned}$$

$$|f_2 f_3| = \frac{\sqrt{7}}{14} J (121) - \frac{3\sqrt{42}}{154} J (141) + \frac{\sqrt{65}}{286} J (161) + \frac{\sqrt{2}}{12} J (231) + \frac{5\sqrt{385}}{462} J (251) - \\ - \frac{\sqrt{3}}{22} J (341) + \frac{\sqrt{910}}{1716} J (361) - \frac{5\sqrt{2310}}{1694} J (451) + \frac{25\sqrt{143}}{9438} J (561)$$

$$(d_6 f_2) d_6 f_3 = \frac{5}{22} J (552)$$

$$|d_1 d_1| = \frac{5\sqrt{33}}{154} J (252) + \frac{5\sqrt{11}}{77} J (452)$$

$$|d_1 f_1| = \frac{\sqrt{330}}{132} J (352) + \frac{5\sqrt{30}}{132} J (552)$$

$$|d_1 f_3| = \frac{5\sqrt{22}}{132} J (352) - \frac{5\sqrt{2}}{132} J (552)$$

$$|d_4 f_3| = -\frac{\sqrt{55}}{33} J (352) + \frac{5\sqrt{5}}{66} J (552)$$

$$|f_4 f_2| = -\frac{\sqrt{55}}{33} J (252) - \frac{\sqrt{165}}{242} J (452) + \frac{\sqrt{10010}}{429} J (562)$$

$$|f_1 f_1| = \frac{\sqrt{33}}{33} J (252) + \frac{5\sqrt{11}}{121} J (452) + \frac{25\sqrt{6006}}{9438} J (562)$$

$$|f_1 f_3| = -\frac{\sqrt{55}}{66} J (252) + \frac{3\sqrt{165}}{242} J (542) - \frac{5\sqrt{10010}}{9438} J (562)$$

$$(d_6 f_2) d_6 f_3 = \frac{5}{18} J (333) - \frac{10\sqrt{11}}{99} J (333) + \frac{10}{99} J (553)$$

$$|d_1 d_2| = -\frac{5\sqrt{14}}{84} J (343) + \frac{5\sqrt{154}}{462} J (453)$$

$$|d_1 f_2| = \frac{10\sqrt{2}}{99} J (533) - \frac{5\sqrt{22}}{198} J (353) - \frac{5\sqrt{2}}{36} J (333)$$

$$|d_2 f_1| = \frac{\sqrt{5}}{18} J (333) - \frac{7\sqrt{55}}{198} J (353) + \frac{5\sqrt{5}}{99} J (553)$$

$$|f_4 f_2| = \frac{\sqrt{35}}{22} J (343) - \frac{5\sqrt{1365}}{1287} J (363) - \frac{\sqrt{385}}{121} J (453) + \frac{10\sqrt{15015}}{14157} J (563)$$

$$|f_1 f_2| = -\frac{\sqrt{35}}{66} J (343) - \frac{5\sqrt{1365}}{858} J (363) + \frac{\sqrt{385}}{363} J (453) + \frac{5\sqrt{15015}}{4719} J (563)$$

$$\begin{aligned}
 (d_1 d_1 | d_1 d_1) &= \frac{1}{2} J(000) + \frac{\sqrt{5}}{7} J(020) - \frac{4}{7} J(040) + \frac{5}{98} J(220) - \frac{4\sqrt{5}}{49} J(240) + \\
 &\quad + \frac{8}{49} J(440) + \frac{15}{98} J(222) + \frac{10\sqrt{3}}{49} J(242) + \frac{10}{49} J(442) \\
 d_1 f_1 &= \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) + \frac{\sqrt{42}}{49} J(120) - \\
 &\quad - \frac{4\sqrt{210}}{245} J(140) + \frac{\sqrt{2}}{42} J(230) - \frac{25\sqrt{154}}{3234} J(250) - \frac{2\sqrt{10}}{105} J(340) + \frac{\sqrt{30}}{42} J(342) + \\
 &\quad + \frac{\sqrt{10}}{28} J(232) + \frac{10\sqrt{770}}{1617} J(450) + \frac{5\sqrt{330}}{462} J(452) + \frac{5\sqrt{110}}{308} J(252) \\
 |d_1 f_2\rangle &= \frac{5\sqrt{6}}{84} J(232) - \frac{5\sqrt{66}}{924} J(252) + \frac{5\sqrt{2}}{42} J(342) - \frac{5\sqrt{22}}{462} J(452) \\
 |d_1 d_2\rangle &= \frac{1}{2} J(000) - \frac{\sqrt{5}}{14} J(020) - \frac{3}{14} J(040) - \frac{5}{49} J(220) + \frac{9\sqrt{5}}{98} J(240) - \frac{2}{49} J(440) \\
 |d_2 f_0\rangle &= -\frac{\sqrt{15}}{21} J(232) + \frac{5\sqrt{165}}{462} J(252) - \frac{2\sqrt{5}}{21} J(342) + \frac{5\sqrt{55}}{231} J(452) \\
 |d_2 f_2\rangle &= \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) + \frac{\sqrt{105}}{98} J(120) - \frac{2\sqrt{21}}{49} J(140) - \\
 &\quad - \frac{5\sqrt{5}}{165} J(230) + \frac{5\sqrt{385}}{3234} J(250) + \frac{4}{21} J(340) - \frac{10\sqrt{77}}{1617} J(450) \\
 |f_0 f_0\rangle &= \frac{1}{2} J(000) + \frac{43\sqrt{5}}{210} J(020) - \frac{1}{77} J(040) + \frac{50\sqrt{13}}{429} J(060) - \frac{43\sqrt{5}}{1155} J(240) + \\
 &\quad + \frac{2}{21} J(220) + \frac{50\sqrt{65}}{3003} J(260) - \frac{12}{77} J(440) - \frac{200\sqrt{13}}{3003} J(460) \\
 |f_1 f_1\rangle &= \frac{1}{2} J(000) + \frac{6\sqrt{5}}{35} J(020) - \frac{37}{154} J(040) + \frac{1}{14} J(220) + \frac{1}{7} J(222) - \\
 &\quad - \frac{39\sqrt{5}}{770} J(240) + \frac{37\sqrt{3}}{231} J(242) - \frac{25\sqrt{65}}{2002} J(260) + \frac{25\sqrt{182}}{2002} J(262) - \\
 &\quad - \frac{2}{77} J(440) + \frac{10}{77} J(442) - \frac{25\sqrt{13}}{286} J(060) + \frac{50\sqrt{13}}{1001} J(460) + \frac{25\sqrt{546}}{3003} J(462) \\
 |f_1 f_2\rangle &= -\frac{\sqrt{15}}{42} J(222) + \frac{5\sqrt{5}}{462} J(242) - \frac{5\sqrt{2730}}{6006} J(262) + \frac{3\sqrt{15}}{77} J(442) - \\
 &\quad - \frac{5\sqrt{910}}{3003} J(462)
 \end{aligned}$$

$$|f_4 f_4\rangle = \frac{1}{2} J(000) + \frac{\sqrt{5}}{14} J(020) - \frac{93}{154} J(040) + \frac{5\sqrt{13}}{143} J(060) - \frac{\sqrt{5}}{22} J(240) + \\ + \frac{5\sqrt{65}}{1001} J(260) + \frac{2}{11} J(440) - \frac{20\sqrt{13}}{1001} J(460)$$

$$|f_5 f_5\rangle = \frac{1}{2} J(000) - \frac{2\sqrt{5}}{21} J(020) - \frac{23}{154} J(040) - \frac{5\sqrt{13}}{858} J(060) - \frac{5}{42} J(220) + \\ + \frac{53\sqrt{5}}{462} J(240) - \frac{5\sqrt{65}}{6006} J(260) - \frac{6}{77} J(440) + \frac{10\sqrt{13}}{3003} J(460)$$

$$(d_1 d_2 | d_1 d_2) = \frac{15}{98} J(221) - \frac{5\sqrt{6}}{98} J(241) + \frac{5}{196} J(441) + \frac{5}{28} J(443)$$

$$|d_1 f_6\rangle = -\frac{3\sqrt{21}}{98} J(121) + \frac{3\sqrt{14}}{196} J(141) + \frac{\sqrt{6}}{42} J(231) + \frac{10\sqrt{1155}}{1617} J(251) - \\ - \frac{1}{42} J(341) - \frac{5\sqrt{770}}{1617} J(451)$$

$$|d_1 f_7\rangle = \frac{3\sqrt{35}}{98} J(121) - \frac{\sqrt{210}}{196} J(141) + \frac{\sqrt{10}}{28} J(231) - \frac{5\sqrt{77}}{539} J(251) - \\ - \frac{\sqrt{15}}{84} J(341) + \frac{5\sqrt{7}}{84} J(343) + \frac{5\sqrt{462}}{5234} J(451) + \frac{5\sqrt{77}}{231} J(453)$$

$$|d_4 f_1\rangle = -\frac{3\sqrt{14}}{196} J(121) + \frac{\sqrt{21}}{196} J(141) + \frac{1}{7} J(231) - \frac{5\sqrt{770}}{2156} J(251) - \\ - \frac{\sqrt{6}}{42} J(341) - \frac{\sqrt{70}}{84} J(343) + \frac{5\sqrt{1155}}{6468} J(451) + \frac{5\sqrt{770}}{924} J(453)$$

$$|d_4 f_2\rangle = \frac{3\sqrt{210}}{196} J(121) - \frac{3\sqrt{35}}{196} J(141) - \frac{\sqrt{15}}{42} J(231) + \frac{5\sqrt{462}}{6468} J(251) + \\ + \frac{\sqrt{10}}{84} J(341) - \frac{5\sqrt{77}}{6468} J(451)$$

$$|f_6 f_1\rangle = \frac{\sqrt{6}}{42} J(221) + \frac{17}{231} J(241) + \frac{25\sqrt{2730}}{6552} J(261) - \frac{5\sqrt{6}}{308} J(441) - \\ - \frac{25\sqrt{435}}{6552} J(461)$$

$$|f_6 f_2\rangle = \frac{5\sqrt{390}}{858} J(463) - \frac{3\sqrt{10}}{44} J(443)$$

$$\begin{aligned} \{f_1, f_2\} &= \frac{\sqrt{10}}{28} J(221) + \frac{13\sqrt{15}}{924} J(241) - \frac{25\sqrt{182}}{4004} J(261) - \frac{\sqrt{10}}{77} J(441) + \\ &+ \frac{\sqrt{10}}{44} J(443) + \frac{25\sqrt{273}}{12012} J(461) + \frac{5\sqrt{390}}{572} J(463) \end{aligned}$$

$$\begin{aligned} \{f_2, f_3\} &= \frac{5\sqrt{6}}{84} J(221) - \frac{145}{924} J(241) + \frac{5\sqrt{2730}}{12012} J(261) + \frac{5\sqrt{6}}{308} J(441) - \\ &- \frac{5\sqrt{455}}{12012} J(461) \end{aligned}$$

$$\begin{aligned} (d_1, f_3) \{d_1, f_3\} &= -\frac{\sqrt{14}}{35} J(131) - \frac{4\sqrt{35}}{77} J(151) + \frac{9}{70} J(111) + \frac{1}{45} J(331) + \\ &+ \frac{4\sqrt{770}}{693} J(351) + \frac{200}{693} J(551) \end{aligned}$$

$$\begin{aligned} \{d_1, f_3\} &= -\frac{3\sqrt{15}}{70} J(111) - \frac{\sqrt{210}}{420} J(131) + \frac{13\sqrt{33}}{231} J(151) + \frac{\sqrt{15}}{90} J(331) + \\ &+ \frac{4\sqrt{462}}{693} J(351) - \frac{20\sqrt{15}}{693} J(551) \end{aligned}$$

$$\begin{aligned} \{d_2, f_3\} &= \frac{3\sqrt{6}}{140} J(111) - \frac{\sqrt{21}}{30} J(131) - \frac{\sqrt{330}}{924} J(151) + \frac{\sqrt{6}}{45} J(331) + \\ &+ \frac{\sqrt{1155}}{198} J(351) - \frac{25\sqrt{6}}{693} J(551) \end{aligned}$$

$$\begin{aligned} \{d_3, f_3\} &= -\frac{9\sqrt{10}}{140} J(111) + \frac{\sqrt{35}}{35} J(131) + \frac{19\sqrt{22}}{308} J(151) - \frac{\sqrt{10}}{90} J(331) - \\ &- \frac{19\sqrt{77}}{1386} J(315) + \frac{5\sqrt{10}}{693} J(551) \end{aligned}$$

$$\begin{aligned} \{f_3, f_3\} &= -\frac{\sqrt{14}}{70} J(121) - \frac{3\sqrt{21}}{154} J(141) - \frac{5\sqrt{130}}{312} J(161) + \frac{1}{45} J(231) + \\ &+ \frac{2\sqrt{770}}{693} J(251) + \frac{\sqrt{6}}{66} J(341) + \frac{5\sqrt{455}}{1404} J(361) + \frac{10\sqrt{1155}}{2541} J(451) + \\ &+ \frac{125\sqrt{286}}{7722} J(561) \end{aligned}$$

$$\{f_3, f_3\} = -\frac{\sqrt{210}}{140} J(121) - \frac{6\sqrt{35}}{385} J(141) + \frac{5\sqrt{78}}{572} J(161) + \frac{\sqrt{15}}{90} J(231) +$$

$$+ \frac{5\sqrt{462}}{693} J(251) + \frac{2\sqrt{10}}{165} J(341) - \frac{5\sqrt{273}}{2574} J(361) + \frac{40\sqrt{77}}{2541} J(451) - \\ - \frac{25\sqrt{4290}}{14157} J(561)$$

$$|f_2 f_2\rangle = -\frac{\sqrt{14}}{28} J(121) + \frac{3\sqrt{21}}{154} J(141) - \frac{\sqrt{130}}{572} J(161) + \frac{1}{18} J(231) + \\ + \frac{5\sqrt{770}}{693} J(251) - \frac{\sqrt{6}}{66} J(341) + \frac{\sqrt{455}}{2374} J(361) - \frac{10\sqrt{1155}}{2541} J(451) + \frac{25\sqrt{286}}{14157} J(561)$$

$$(d_1 f_1 | d_1 f_1) = \frac{12}{35} J(110) + \frac{4\sqrt{21}}{165} J(130) - \frac{20\sqrt{33}}{231} J(150) + \frac{1}{45} J(330) + \frac{1}{12} J(332) - \\ - \frac{10\sqrt{77}}{693} J(350) + \frac{5\sqrt{11}}{66} J(352) + \frac{125}{693} J(550) + \frac{25}{132} J(552)$$

$$|d_1 f_2\rangle = \frac{\sqrt{15}}{36} J(332) + \frac{\sqrt{165}}{99} J(352) - \frac{5\sqrt{15}}{396} J(552)$$

$$|d_2 d_2\rangle = \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) - \frac{2\sqrt{42}}{49} J(120) + \\ + \frac{\sqrt{210}}{245} J(140) - \frac{\sqrt{2}}{21} J(230) + \frac{25\sqrt{154}}{1617} J(250) + \frac{\sqrt{10}}{210} J(340) - \frac{5\sqrt{770}}{3234} J(450)$$

$$|d_2 f_2\rangle = -\frac{\sqrt{6}}{18} J(332) - \frac{5\sqrt{66}}{396} J(352) + \frac{25\sqrt{6}}{396} J(552)$$

$$|d_3 f_2\rangle = \frac{3\sqrt{10}}{35} J(110) - \frac{\sqrt{210}}{70} J(130) - \frac{\sqrt{330}}{154} J(150) - \frac{\sqrt{10}}{45} J(330) + \\ + \frac{\sqrt{770}}{126} J(350) - \frac{25\sqrt{10}}{1386} J(550)$$

$$|f_2 f_2\rangle = \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) + \frac{4\sqrt{42}}{105} J(120) + \\ + \frac{6\sqrt{210}}{385} J(140) + \frac{20\sqrt{2730}}{3093} J(160) + \frac{2\sqrt{2}}{45} J(230) - \frac{10\sqrt{154}}{693} J(250) +$$

$$+ \frac{\sqrt{10}}{55} J(340) + \frac{10\sqrt{130}}{1287} J(360) - \frac{5\sqrt{770}}{847} J(450) - \frac{250\sqrt{10010}}{99099} J(560)$$

$$|f_2 f_2\rangle = -\frac{\sqrt{6}}{18} J(232) - \frac{5\sqrt{66}}{198} J(252) - \frac{\sqrt{2}}{44} J(342) - \frac{\sqrt{273}}{117} J(362) - \\ - \frac{5\sqrt{22}}{484} J(452) + \frac{5\sqrt{3003}}{1287} J(562)$$

$$\begin{aligned} |f_1 f_2| &= \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) + \frac{\sqrt{42}}{35} J(120) + \\ &+ \frac{\sqrt{210}}{385} J(140) - \frac{25\sqrt{2730}}{1001} J(160) + \frac{\sqrt{2}}{30} J(230) - \frac{5\sqrt{154}}{462} J(250) + \\ &+ \frac{\sqrt{10}}{30} J(232) + \frac{\sqrt{110}}{66} J(252) + \frac{\sqrt{10}}{330} J(340) + \frac{\sqrt{30}}{66} J(342) - \frac{5\sqrt{130}}{858} J(360) + \\ &+ \frac{5\sqrt{455}}{858} J(362) - \frac{5\sqrt{770}}{5082} J(450) + \frac{5\sqrt{330}}{726} J(452) + \frac{125\sqrt{10010}}{66066} J(560) + \\ &+ \frac{25\sqrt{5005}}{9438} J(562) \end{aligned}$$

$$\begin{aligned} |f_1 f_3| &= -\frac{\sqrt{6}}{36} J(232) - \frac{5\sqrt{66}}{396} J(252) + \frac{3\sqrt{2}}{44} J(342) - \frac{5\sqrt{273}}{2574} J(362) + \\ &+ \frac{15\sqrt{22}}{484} J(452) - \frac{25\sqrt{3003}}{28314} J(562) \end{aligned}$$

$$\begin{aligned} |f_2 f_3| &= \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) - \frac{\sqrt{210}}{55} J(140) + \\ &+ \frac{2\sqrt{2730}}{1001} J(160) - \frac{7\sqrt{10}}{330} J(340) + \frac{\sqrt{130}}{429} J(360) + \frac{5\sqrt{770}}{726} J(450) - \\ &- \frac{25\sqrt{10010}}{33033} J(560) \end{aligned}$$

$$\begin{aligned} |f_3 f_4| &= \frac{\sqrt{210}}{35} J(010) + \frac{\sqrt{10}}{30} J(030) - \frac{5\sqrt{770}}{462} J(050) - \frac{\sqrt{42}}{21} J(120) + \\ &+ \frac{3\sqrt{210}}{385} J(140) - \frac{\sqrt{2730}}{3003} J(160) - \frac{\sqrt{2}}{18} J(230) + \frac{25\sqrt{154}}{1386} J(250) + \\ &+ \frac{\sqrt{10}}{110} J(340) - \frac{\sqrt{130}}{2574} J(360) - \frac{5\sqrt{770}}{1694} J(450) + \frac{25\sqrt{10010}}{198198} J(560) \end{aligned}$$

$$\begin{aligned} |d_1 f_4| d_1 f_5 &= \frac{3}{14} J(111) + \frac{\sqrt{14}}{14} J(131) - \frac{2\sqrt{55}}{77} J(151) + \frac{1}{12} J(331) + \frac{5}{36} J(333) - \\ &- \frac{\sqrt{770}}{231} J(351) + \frac{10\sqrt{11}}{99} J(353) + \frac{10}{231} J(551) + \frac{20}{99} J(553) \end{aligned}$$

$$\begin{aligned} |d_2 f_4| &= -\frac{3\sqrt{10}}{140} J(111) + \frac{3\sqrt{35}}{140} J(131) - \frac{3\sqrt{22}}{308} J(151) + \frac{\sqrt{10}}{30} J(331) - \\ &- \frac{13\sqrt{77}}{924} J(351) - \frac{\sqrt{10}}{36} J(333) + \frac{\sqrt{110}}{396} J(353) + \frac{5\sqrt{10}}{462} J(551) + \frac{5\sqrt{10}}{99} J(553) \end{aligned}$$

$$|d_2 f_2| = \frac{3\sqrt{6}}{28} J(111) + \frac{\sqrt{21}}{84} J(131) - \frac{5\sqrt{330}}{921} J(151) - \frac{\sqrt{6}}{36} J(331) + \\ + \frac{5\sqrt{1155}}{2772} J(351) - \frac{5\sqrt{6}}{1386} J(551)$$

$$|f_6 f_2| = \frac{\sqrt{210}}{210} J(121) + \frac{3\sqrt{35}}{154} J(141) + \frac{25\sqrt{78}}{936} J(161) + \frac{\sqrt{15}}{90} J(231) - \\ - \frac{\sqrt{462}}{693} J(251) + \frac{\sqrt{10}}{44} J(341) + \frac{25\sqrt{273}}{2808} J(361) - \frac{5\sqrt{77}}{847} J(451) - \frac{25\sqrt{4290}}{15444} J(561)$$

$$|f_6 f_4| = -\frac{\sqrt{70}}{44} J(343) + \frac{5\sqrt{2730}}{2574} J(363) - \frac{\sqrt{770}}{121} J(453) + \frac{10\sqrt{30030}}{14157} J(563)$$

$$|f_7 f_2| = \frac{\sqrt{14}}{28} J(121) + \frac{2\sqrt{21}}{77} J(141) - \frac{5\sqrt{130}}{572} J(161) + \frac{1}{12} J(231) -$$

$$- \frac{\sqrt{770}}{462} J(251) + \frac{\sqrt{6}}{33} J(341) - \frac{5\sqrt{455}}{1716} J(361) - \frac{4\sqrt{1155}}{2541} J(451) + \\ + \frac{5\sqrt{2730}}{1716} J(363) + \frac{\sqrt{70}}{132} J(343) + \frac{\sqrt{770}}{363} J(453) + \frac{5\sqrt{30030}}{4719} J(563) + \frac{25\sqrt{286}}{9438} J(561)$$

$$|f_7 f_4| = \frac{\sqrt{210}}{84} J(121) - \frac{3\sqrt{35}}{154} J(141) + \frac{5\sqrt{78}}{1716} J(161) + \frac{\sqrt{15}}{36} J(231) -$$

$$- \frac{5\sqrt{462}}{1386} J(251) - \frac{\sqrt{10}}{44} J(341) + \frac{5\sqrt{273}}{5148} J(361) + \frac{5\sqrt{77}}{847} J(451) - \frac{5\sqrt{4290}}{28314} J(561)$$

$$(d_1 f_4 | d_1 f_2) = \frac{5}{36} J(332) - \frac{5\sqrt{11}}{198} J(352) + \frac{5}{396} J(552) + \frac{5}{33} J(554)$$

$$|d_2 d_4| = \frac{5\sqrt{462}}{462} J(454)$$

$$|d_4 f_4| = -\frac{\sqrt{10}}{18} J(332) + \frac{7\sqrt{110}}{396} J(352) - \frac{5\sqrt{10}}{396} J(552)$$

$$|d_2 f_4| = \frac{5}{33} J(554)$$

$$|f_6 f_4| = -\frac{\sqrt{10}}{18} J(232) + \frac{\sqrt{110}}{198} J(252) - \frac{\sqrt{30}}{132} J(342) + \frac{\sqrt{455}}{117} J(362) + \\ + \frac{\sqrt{330}}{1452} J(452) - \frac{\sqrt{5005}}{1287} J(562)$$

$$|f_1 f_1\rangle = \frac{\sqrt{6}}{18} J(232) - \frac{\sqrt{66}}{198} J(252) + \frac{5\sqrt{2}}{66} J(342) + \frac{25\sqrt{273}}{2574} J(362) - \\ - \frac{5\sqrt{22}}{726} J(452) - \frac{25\sqrt{3003}}{28314} J(562)$$

$$|f_1 f_2\rangle = -\frac{\sqrt{10}}{36} J(232) + \frac{\sqrt{110}}{396} J(252) + \frac{\sqrt{30}}{44} J(342) - \frac{5\sqrt{455}}{2574} J(362) - \\ - \frac{\sqrt{330}}{484} J(452) - \frac{\sqrt{770}}{242} J(454) + \frac{5\sqrt{5095}}{28314} J(562) + \frac{25\sqrt{2902}}{9438} J(564)$$

$$|f_2 f_2\rangle = \frac{5\sqrt{462}}{726} J(454) + \frac{5\sqrt{30030}}{4719} J(564)$$

$$(d_2 d_2 | d_2 d_2) = -\frac{2\sqrt{5}}{7} J(020) + \frac{1}{7} J(040) + \frac{1}{2} J(000) + \frac{10}{49} J(220) - \frac{2\sqrt{5}}{49} J(240) + \\ + \frac{1}{98} J(440) + \frac{5}{14} J(444)$$

$$|d_2 f_2\rangle = \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) - \frac{\sqrt{105}}{49} J(120) + \frac{\sqrt{21}}{98} J(140) + \\ + \frac{2\sqrt{5}}{21} J(230) - \frac{5\sqrt{385}}{1617} J(250) - \frac{1}{21} J(340) + \frac{5\sqrt{77}}{3234} J(450) + \frac{5\sqrt{462}}{462} J(454)$$

$$|f_2 f_2\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{105} J(020) + \frac{53}{154} J(040) + \frac{50\sqrt{13}}{429} J(060) - \frac{4}{21} J(220) - \\ - \frac{68\sqrt{5}}{1155} J(240) - \frac{100\sqrt{65}}{3903} J(260) + \frac{3}{77} J(440) + \frac{50\sqrt{13}}{3003} J(460)$$

$$|f_1 f_1\rangle = \frac{1}{2} J(000) - \frac{3\sqrt{5}}{70} J(020) + \frac{9}{77} J(040) - \frac{25\sqrt{13}}{286} J(060) - \frac{1}{7} J(220) + \\ + \frac{\sqrt{5}}{770} J(240) + \frac{25\sqrt{65}}{1001} J(260) + \frac{1}{154} J(440) - \frac{25\sqrt{13}}{2002} J(460)$$

$$|f_1 f_2\rangle = -\frac{\sqrt{15}}{22} J(444) + \frac{25\sqrt{39}}{858} J(464)$$

$$|f_2 f_2\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{7} J(020) - \frac{19}{77} J(040) + \frac{5\sqrt{13}}{143} J(060) + \frac{\sqrt{5}}{11} J(240) - \\ - \frac{10\sqrt{65}}{1001} J(260) - \frac{1}{22} J(440) + \frac{5}{22} J(444) + \frac{5\sqrt{13}}{1001} J(460) + \frac{5\sqrt{65}}{143} J(464)$$

$$\begin{aligned} |f_2 f_2\rangle &= \frac{1}{2} J(000) + \frac{13\sqrt{5}}{42} J(020) + \frac{16}{77} J(040) - \frac{5\sqrt{13}}{858} J(060) + \frac{5}{21} J(220) - \\ &- \frac{29\sqrt{5}}{462} J(240) + \frac{5\sqrt{65}}{3063} J(260) + \frac{3}{154} J(440) - \frac{5\sqrt{13}}{6906} J(460) \end{aligned}$$

$$(d_2 f_2 | d_2 f_2) = \frac{2}{9} J(332) - \frac{10\sqrt{11}}{99} J(352) + \frac{25}{198} J(552)$$

$$\begin{aligned} |f_2 f_2\rangle &= \frac{2}{9} J(232) - \frac{5\sqrt{11}}{99} J(252) + \frac{\sqrt{3}}{33} J(342) - \frac{2\sqrt{182}}{117} J(362) - \\ &- \frac{5\sqrt{33}}{726} J(452) + \frac{5\sqrt{2002}}{1287} J(562) \end{aligned}$$

$$\begin{aligned} |f_2 f_2\rangle &= -\frac{2\sqrt{15}}{45} J(232) + \frac{\sqrt{165}}{99} J(252) - \frac{2\sqrt{5}}{33} J(342) - \frac{5\sqrt{2730}}{1287} J(362) + \\ &+ \frac{5\sqrt{55}}{363} J(452) + \frac{25\sqrt{30030}}{28314} J(562) \end{aligned}$$

$$\begin{aligned} |f_2 f_2\rangle &= \frac{1}{9} J(232) - \frac{5\sqrt{11}}{198} J(252) - \frac{\sqrt{3}}{11} J(342) + \frac{5\sqrt{182}}{1287} J(362) + \\ &+ \frac{5\sqrt{33}}{242} J(452) - \frac{25\sqrt{2002}}{28314} J(562) \end{aligned}$$

$$\begin{aligned} (d_2 f_2 | d_2 f_2) &= \frac{3}{140} J(111) - \frac{\sqrt{14}}{35} J(131) + \frac{\sqrt{55}}{154} J(151) + \frac{2}{15} J(331) + \frac{1}{18} J(333) - \\ &- \frac{\sqrt{770}}{231} J(351) - \frac{5\sqrt{11}}{99} J(353) + \frac{25}{924} J(551) + \frac{25}{198} J(553) \end{aligned}$$

$$\begin{aligned} |d_2 f_2\rangle &= -\frac{3\sqrt{15}}{140} J(111) + \frac{\sqrt{210}}{60} J(131) - \frac{4\sqrt{33}}{231} J(151) - \frac{\sqrt{15}}{45} J(331) + \\ &+ \frac{\sqrt{462}}{396} J(351) - \frac{5\sqrt{15}}{2772} J(551) \end{aligned}$$

$$\begin{aligned} |f_2 f_2\rangle &= -\frac{\sqrt{21}}{210} J(121) - \frac{3\sqrt{14}}{308} J(141) - \frac{5\sqrt{195}}{936} J(161) + \frac{\sqrt{6}}{45} J(231) - \\ &- \frac{\sqrt{1155}}{1386} J(251) + \frac{1}{11} J(341) + \frac{5\sqrt{2730}}{1404} J(361) - \frac{5\sqrt{770}}{3388} J(451) - \frac{125\sqrt{429}}{30888} J(561) \end{aligned}$$

$$|f_2 f_2\rangle = \frac{\sqrt{7}}{22} J(343) - \frac{5\sqrt{273}}{1287} J(363) - \frac{5\sqrt{77}}{242} J(453) + \frac{25\sqrt{3063}}{14157} J(563)$$

$$\begin{aligned}
 |f_1 f_2| &= -\frac{\sqrt{35}}{140} J(121) - \frac{\sqrt{210}}{385} J(141) + \frac{5\sqrt{13}}{572} J(161) + \frac{\sqrt{10}}{30} J(231) - \\
 &- \frac{5\sqrt{77}}{924} J(251) + \frac{4\sqrt{15}}{165} J(341) - \frac{\sqrt{7}}{66} J(343) - \frac{5\sqrt{182}}{838} J(361) - \frac{5\sqrt{273}}{858} J(363) - \\
 &- \frac{5\sqrt{462}}{2541} J(451) + \frac{5\sqrt{77}}{726} J(453) + \frac{25\sqrt{715}}{18876} J(561) + \frac{25\sqrt{3003}}{9438} J(563) \\
 |f_2 f_3| &= -\frac{\sqrt{21}}{84} J(121) + \frac{3\sqrt{14}}{368} J(141) - \frac{\sqrt{195}}{1716} J(161) + \frac{\sqrt{6}}{18} J(231) - \\
 &- \frac{5\sqrt{1155}}{2772} J(251) - \frac{1}{11} J(341) + \frac{\sqrt{2730}}{2574} J(361) + \frac{5\sqrt{770}}{3388} J(451) - \frac{25\sqrt{429}}{56628} J(561) \\
 (d_2 f_1 d_2 f_2) &= \frac{3}{14} J(110) - \frac{2\sqrt{21}}{21} J(130) - \frac{5\sqrt{33}}{231} J(150) + \frac{2}{9} J(330) - \\
 &- \frac{10\sqrt{77}}{693} J(350) + \frac{25}{1386} J(550) + \frac{5}{33} J(554) \\
 |f_3 f_4| &= \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) + \frac{2\sqrt{105}}{105} J(120) + \frac{3\sqrt{21}}{77} J(140) + \\
 &+ \frac{50\sqrt{273}}{3003} J(160) - \frac{4\sqrt{5}}{45} J(230) + \frac{2\sqrt{385}}{693} J(250) - \frac{2}{11} J(340) - \frac{100\sqrt{13}}{1287} J(360) + \\
 &+ \frac{5\sqrt{77}}{847} J(450) + \frac{250\sqrt{1001}}{99099} J(560) \\
 |f_1 f_3| &= \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) + \frac{\sqrt{105}}{70} J(120) + \frac{\sqrt{21}}{154} J(140) - \\
 &- \frac{25\sqrt{273}}{2002} J(160) - \frac{\sqrt{5}}{15} J(230) + \frac{\sqrt{385}}{462} J(250) - \frac{1}{33} J(340) + \frac{25\sqrt{13}}{429} J(360) + \\
 &+ \frac{5\sqrt{77}}{5082} J(450) - \frac{125\sqrt{1001}}{60066} J(560) \\
 |f_1 f_4| &= -\frac{\sqrt{770}}{242} J(454) + \frac{25\sqrt{2002}}{9438} J(564) \\
 |f_2 f_3| &= \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) - \frac{\sqrt{21}}{22} J(140) + \\
 &+ \frac{5\sqrt{273}}{1001} J(160) + \frac{7}{33} J(340) - \frac{10\sqrt{13}}{429} J(360) - \frac{5\sqrt{77}}{726} J(450) + \\
 &+ \frac{5\sqrt{462}}{726} J(454) + \frac{25\sqrt{1001}}{33033} J(560) + \frac{5\sqrt{30030}}{4719} J(564)
 \end{aligned}$$

$$\begin{aligned} |f_3 f_4| &= \frac{\sqrt{21}}{14} J(010) - \frac{1}{3} J(030) + \frac{5\sqrt{77}}{462} J(050) - \frac{\sqrt{105}}{42} J(120) + \\ &+ \frac{3\sqrt{21}}{154} J(140) - \frac{5\sqrt{273}}{6006} J(160) + \frac{\sqrt{5}}{9} J(230) - \frac{5\sqrt{385}}{1386} J(250) - \\ &- \frac{1}{11} J(340) + \frac{5\sqrt{13}}{1287} J(360) + \frac{5\sqrt{77}}{1694} J(450) - \frac{25\sqrt{1001}}{198198} J(560) \end{aligned}$$

$$\begin{aligned} (d_1 f_4 | d_2 f_3) &= \frac{9}{28} J(111) - \frac{\sqrt{14}}{14} J(131) + \frac{\sqrt{55}}{154} J(151) + \frac{1}{18} J(331) - \\ &- \frac{\sqrt{770}}{1386} J(351) + \frac{5}{2772} J(351) + \frac{25}{78} J(555) \end{aligned}$$

$$\begin{aligned} |f_4 f_1| &= \frac{\sqrt{35}}{70} J(121) + \frac{3\sqrt{210}}{308} J(141) + \frac{25\sqrt{13}}{312} J(161) - \frac{\sqrt{10}}{90} J(231) + \\ &+ \frac{\sqrt{77}}{1386} J(251) - \frac{\sqrt{15}}{66} J(341) - \frac{25\sqrt{182}}{2808} J(361) + \frac{5\sqrt{462}}{10164} J(451) + \\ &+ \frac{25\sqrt{715}}{30888} J(561) \end{aligned}$$

$$\begin{aligned} |f_1 f_2| &= \frac{\sqrt{21}}{28} J(121) + \frac{3\sqrt{14}}{77} J(141) - \frac{5\sqrt{195}}{572} J(161) - \frac{\sqrt{6}}{36} J(231) + \\ &+ \frac{\sqrt{1155}}{2772} J(251) - \frac{2}{33} J(341) + \frac{5\sqrt{2730}}{5148} J(361) + \frac{\sqrt{770}}{2541} J(451) - \frac{25\sqrt{429}}{56628} J(561) \end{aligned}$$

$$\begin{aligned} |f_2 f_3| &= \frac{\sqrt{35}}{28} J(121) - \frac{3\sqrt{210}}{308} J(141) + \frac{5\sqrt{13}}{572} J(161) - \frac{\sqrt{10}}{36} J(231) + \\ &+ \frac{5\sqrt{77}}{2772} J(251) + \frac{\sqrt{15}}{66} J(341) - \frac{5\sqrt{182}}{5148} J(361) - \frac{5\sqrt{462}}{10164} J(451) + \\ &+ \frac{5\sqrt{715}}{56628} J(561) + \frac{25\sqrt{77}}{858} J(565) \end{aligned}$$

$$\begin{aligned} (f_1 f_4 | f_3 f_5) &= \frac{1}{2} J(000) + \frac{4\sqrt{5}}{15} J(020) + \frac{6}{11} J(040) + \frac{100\sqrt{13}}{429} J(060) + \frac{8}{45} J(220) + \\ &+ \frac{8\sqrt{5}}{55} J(240) + \frac{80\sqrt{65}}{1287} J(260) + \frac{18}{121} J(440) + \frac{200\sqrt{13}}{1573} J(460) + \frac{5000}{14137} J(660) \end{aligned}$$

$$\begin{aligned} |f_1 f_5| &= \frac{1}{2} J(000) + \frac{7\sqrt{5}}{30} J(020) + \frac{7}{22} J(040) + \frac{25\sqrt{13}}{858} J(060) + \frac{2}{15} J(220) + \\ &+ \frac{\sqrt{5}}{15} J(240) + \frac{3}{121} J(440) - \frac{175\sqrt{13}}{4719} J(460) - \frac{1250}{4719} J(660) \end{aligned}$$

$$|f_3 f_3\rangle = \frac{1}{2} J(000) + \frac{2\sqrt{5}}{15} J(020) - \frac{1}{22} J(040) + \frac{5\sqrt{13}}{429} J(060) - \\ - \frac{14\sqrt{5}}{165} J(240) - \frac{21}{121} J(440) + \frac{4\sqrt{65}}{429} J(260) - \frac{260\sqrt{13}}{4719} J(460) + \frac{500}{4719} J(660)$$

$$|f_3 f_3\rangle = \frac{1}{2} J(000) - \frac{\sqrt{5}}{30} J(020) + \frac{9}{22} J(040) + \frac{95\sqrt{13}}{858} J(060) - \frac{2}{9} J(220) - \\ - \frac{3\sqrt{5}}{55} J(240) - \frac{52\sqrt{65}}{1287} J(260) + \frac{9}{121} J(440) + \frac{45\sqrt{13}}{1573} J(460) - \frac{250}{14157} J(660)$$

$$\langle f_6 f_1 | f_6 f_1 \rangle = \frac{1}{45} J(221) + \frac{\sqrt{6}}{33} J(241) + \frac{5\sqrt{455}}{702} J(261) + \frac{15}{242} J(441) + \\ + \frac{25\sqrt{2730}}{5148} J(461) + \frac{4375}{16848} J(661)$$

$$\langle f_1 f_2 | f_1 f_2 \rangle = \frac{\sqrt{15}}{90} J(221) + \frac{23\sqrt{10}}{660} J(241) + \frac{215\sqrt{273}}{30888} J(261) + \frac{2\sqrt{15}}{121} J(441) + \\ + \frac{325\sqrt{182}}{56628} J(461) - \frac{875\sqrt{15}}{30888} J(661)$$

$$\langle f_3 f_3 | f_3 f_3 \rangle = \frac{1}{18} J(221) + \frac{\sqrt{6}}{44} J(241) - \frac{287\sqrt{455}}{30888} J(261) - \frac{15}{242} J(441) - \\ - \frac{245\sqrt{2730}}{113256} J(461) + \frac{875}{30888} J(661)$$

$$\langle f_6 f_2 | f_6 f_2 \rangle = \frac{2}{9} J(222) + \frac{2\sqrt{3}}{33} J(242) - \frac{4\sqrt{182}}{117} J(262) + \frac{3}{242} J(442) - \\ - \frac{2\sqrt{546}}{429} J(462) + \frac{28}{117} J(662)$$

$$\langle f_1 f_1 | f_1 f_1 \rangle = -\frac{2\sqrt{15}}{45} J(222) - \frac{13\sqrt{5}}{165} J(242) - \frac{\sqrt{2730}}{2145} J(262) - \frac{\sqrt{15}}{121} J(442) + \\ + \frac{29\sqrt{910}}{9438} J(462) + \frac{70\sqrt{15}}{1287} J(662)$$

$$\langle f_1 f_3 | f_1 f_3 \rangle = \frac{1}{9} J(222) - \frac{5\sqrt{3}}{66} J(242) - \frac{2\sqrt{182}}{429} J(262) - \frac{9}{242} J(442) + \\ + \frac{71\sqrt{546}}{9438} J(462) - \frac{70}{1287} J(662)$$

$$(f_3 f_3 | f_3 f_3) = \frac{63}{242} J(443) - \frac{210 \sqrt{39}}{4719} J(463) + \frac{350}{4719} J(663)$$

$$|f_1 f_1) = -\frac{21}{242} J(443) - \frac{245 \sqrt{39}}{9438} J(463) + \frac{175}{1573} J(663)$$

$$\begin{aligned} (f_1 f_1 | f_1 f_1) &= \frac{1}{2} J(000) + \frac{\sqrt{5}}{5} J(020) + \frac{1}{11} J(040) - \frac{25 \sqrt{13}}{143} J(060) + \frac{1}{10} J(220) + \\ &+ \frac{2}{15} J(222) + \frac{\sqrt{5}}{55} J(240) + \frac{4 \sqrt{3}}{33} J(242) - \frac{5 \sqrt{65}}{143} J(260) + \frac{10 \sqrt{182}}{429} J(262) + \\ &+ \frac{1}{242} J(440) + \frac{10}{121} J(442) - \frac{25 \sqrt{13}}{1573} J(460) + \frac{625}{3146} J(660) + \\ &+ \frac{50 \sqrt{546}}{4719} J(462) + \frac{875}{4719} J(662) \end{aligned}$$

$$\begin{aligned} |f_1 f_3) &= -\frac{\sqrt{15}}{45} J(222) + \frac{4 \sqrt{5}}{165} J(242) - \frac{7 \sqrt{2730}}{2574} J(262) + \frac{3 \sqrt{15}}{121} J(442) + \\ &+ \frac{35 \sqrt{910}}{9438} J(462) - \frac{175 \sqrt{15}}{14157} J(662) \end{aligned}$$

$$\begin{aligned} |f_2 f_2) &= \frac{1}{2} J(000) + \frac{\sqrt{5}}{10} J(020) - \frac{3}{11} J(040) - \frac{15 \sqrt{13}}{286} J(060) - \frac{7 \sqrt{5}}{110} J(240) - \\ &- \frac{7}{242} J(440) + \frac{\sqrt{65}}{143} J(260) + \frac{185 \sqrt{13}}{3146} J(460) - \frac{125}{1573} J(660) \end{aligned}$$

$$\begin{aligned} |f_2 f_3) &= \frac{1}{2} J(000) - \frac{\sqrt{5}}{15} J(020) + \frac{2}{11} J(040) - \frac{40 \sqrt{13}}{429} J(060) - \frac{1}{6} J(220) + \\ &+ \frac{2 \sqrt{5}}{165} J(240) + \frac{4 \sqrt{65}}{143} J(260) + \frac{3}{242} J(440) + \frac{125}{9438} J(660) - \frac{115 \sqrt{13}}{4719} J(460) \end{aligned}$$

$$\begin{aligned} (f_1 f_2 | f_1 f_2) &= \frac{1}{12} J(221) + \frac{2 \sqrt{6}}{33} J(241) - \frac{5 \sqrt{455}}{858} J(261) + \frac{8}{121} J(441) - \\ &- \frac{10 \sqrt{2730}}{4719} J(461) + \frac{7}{242} J(443) + \frac{35 \sqrt{39}}{1573} J(463) + \frac{875}{18876} J(661) + \frac{525}{3146} J(663) \end{aligned}$$

$$\begin{aligned} |f_2 f_3) &= \frac{\sqrt{15}}{36} J(221) + \frac{\sqrt{10}}{132} J(241) - \frac{5 \sqrt{273}}{1287} J(261) - \frac{2 \sqrt{15}}{121} J(441) + \\ &+ \frac{95 \sqrt{182}}{18876} J(461) - \frac{175 \sqrt{15}}{56628} J(661) \end{aligned}$$

$$\begin{aligned} (f_1 f_2 | f_1 f_2) &= \frac{1}{18} J (222) - \frac{\sqrt{3}}{11} J (242) + \frac{5\sqrt{182}}{1287} J (262) + \frac{27}{242} J (442) - \\ &- \frac{5\sqrt{546}}{1573} J (462) + \frac{21}{242} J (444) - \frac{35\sqrt{65}}{1573} J (464) + \frac{175}{14137} J (662) + \\ &+ \frac{875}{9438} J (664) \end{aligned}$$

$$(f_2 f_2) = -\frac{7\sqrt{15}}{242} J (444) - \frac{35\sqrt{39}}{9438} J (464) + \frac{175\sqrt{15}}{4719} J (664)$$

$$\begin{aligned} (f_1 f_2 | f_2 f_2) &= \frac{1}{2} J (000) - \frac{7}{11} J (040) + \frac{10\sqrt{13}}{143} J (060) + \frac{49}{242} J (440) + \frac{35}{242} J (444) - \\ &- \frac{70\sqrt{13}}{1573} J (460) + \frac{70\sqrt{65}}{1573} J (464) + \frac{50}{1573} J (660) + \frac{350}{1573} J (664) \end{aligned}$$

$$\begin{aligned} (f_1 f_2) &= \frac{1}{2} J (000) - \frac{\sqrt{5}}{6} J (020) - \frac{2}{11} J (040) + \frac{25\sqrt{13}}{838} J (060) + \frac{7\sqrt{5}}{66} J (240) - \\ &- \frac{5\sqrt{65}}{429} J (260) - \frac{21}{242} J (440) + \frac{125\sqrt{13}}{9438} J (460) - \frac{25}{4719} J (660) \end{aligned}$$

$$\begin{aligned} (f_2 f_2 | f_2 f_2) &= \frac{5}{36} J (221) - \frac{5\sqrt{6}}{66} J (241) + \frac{5\sqrt{455}}{2574} J (261) + \frac{15}{242} J (441) - \\ &- \frac{5\sqrt{2730}}{9438} J (461) + \frac{175}{56628} J (661) + \frac{1925}{9438} J (665) \end{aligned}$$

$$\begin{aligned} (f_2 f_2 | f_1 f_2) &= \frac{1}{2} J (000) - \frac{\sqrt{5}}{3} J (020) + \frac{3}{11} J (040) - \frac{5\sqrt{13}}{429} J (060) + \frac{5}{18} J (220) - \\ &- \frac{\sqrt{5}}{11} J (240) + \frac{5\sqrt{65}}{1287} J (260) + \frac{9}{242} J (440) - \frac{5\sqrt{13}}{1573} J (460) + \frac{25}{28314} J (660) + \\ &+ \frac{175}{429} J (666) \end{aligned}$$