

Rendiconti Accademia Nazionale delle Scienze detta dei XL *Memorie di Scienze Fisiche e Naturali* 126° (2008), Vol. XXXII, P. II, pp. 127-147

GIUSEPPE BASINI* - SALVATORE CAPOZZIELLO**

On the possibility of travelling in time in open quantum relativity***

Abstract – The time travel possibility is taking new strength starting from some relativistic theories and particularly from Open Quantum Relativity, which connects Quantum Mechanics and General Relativity, in a comprehensive symplectic structure. In this framework, the existence of a five dimensional space and of two time arrows come out as a natural necessity under the general assumption that the conservation laws can never be violated. The comparison between the forecast of this dynamical theory and the experimental data of the most recent observations, in several fields of physics, is really satisfactory, and provides a solid test-bed for the hypothesis of two Time arrows. The existence of such a feature, if confirmed, will change dramatically any conception on Time travels.

1. INTRODUCTION

General Relativity is the cornerstone of a new way to see the Nature and opened the doors to a different way to define also old concepts, first of all Time. The most surprising result is the possibility, conceivable also in standard Relativity, of relativistic time travels in a "Relative Future", as we will recall below. Beside this fact, it is in relativistic theories that the Time travel hypothesis can eventually get even more relevant meaning. Let us first remember the four classes of possibilities that are to be taken into account dealing whit Time and Time Machine: 1) travels in time forbidden by conjectures (like Hawking's Chronological Protection) which seem really reasonable, since based on the common sense which is observing the

^{*} Laboratori Nazionali di Frascati, INFN. Frascati (Roma).

^{**} Dipartimento di Scienze Fisiche, Università di Napoli Federico II, INFN sez. di Napoli. E-mail: capozziello@na.infn.it

^{***} Memoria presentata dal socio Maurizio Cumo.

apparent lack of time voyagers, but also conjectures not mathematically well founded, because coming out from semi-classical approximations, which are inadequate to a quantum world); 2) hypotheses exotically based on particles, the tachyons, not measurables and very difficult to define without violating Special and General Relativity prescriptions; 3) time travels of more general meaning than standard, but still allowed in the rigorous limit of General Relativity with, up to now, the great difficulty to solve several paradoxes seriously questioning their own existence; 4) and, finally, time travels coming out from relativistic theories, like Open Quantum Relativity (OQR) [1], in which two time arrows and Closed Time-like Curves (CTCs) are intrinsic features of Nature and whose possible effects are the topic of this paper. A general theory which certainly represents a breaking point with respect to traditional points of view, but able to reconcile Quantum Mechanics and Causality in a framework where Relativity and Quantum Mechanics are deeply connected.

In fact, OQR is a coherent dynamical theory, able to include in its scheme new phenomena and experimental observations, that, up to now, have been very difficult to include in the standard pictures, mainly because of the difficulty in defining a unified picture of Quantum Mechanics and Relativity.

This theory is based on a principle: the General Conservation Principle; on a method: the dynamical emergence of the physical laws from a unitary frame with the minimal number of free variables and, finally, on a fundamental consideration: the possibility to achieve a covariant symplectic structure for both Quantum Mechanics and General Relativity. The need of a new general unifying theory comes out from the observation that there are too many open questions, in today physics, and at a very fundamental level. Namely: a) the paradox pointed out by Einstein-Podolsky-Rosen (EPR) [2], i.e. the possibility, under particular conditions, to perturb a physical object without interacting with it in any known way; b) The standard cosmological model and the Big Bang theory, still not satisfactory addressed due to shortcomings like the initial singularity, the dark energy and the dark matter [3]; c) the existence of Black Holes, which, despite several attempts [4, 5], seem to violate the mass-energy conservation [6]; d) the consideration of quarks as elementary particles, even if it is generally hypothesized that they can never get their individuality [7]; e) the absence of observed primary antimatter in our universe, despite of the standard creation of matter and antimatter always in symmetric pairs [8, 9, 10, 11]; f) the experimental results in quantum teleportation [12, 13] which suggest a $\Delta T = 0$ in transfer information time, in that way questioning the relativity; g) the lack of unitary description of all fundamental interactions [7]; h) and finally the fundamental question of the absence of a unitary theory [14, 15] connecting Quantum Mechanics (QM) and Relativity. In our opinion, instead of suggesting approaches which need a very high number of dimensions or introduce "ad hoc" hypotheses like the spontaneous symmetry breaking, a straightforward way to solve the shortcomings of modern physics is a new approach, deeply related

to conservation laws, because we propose the following point of view: consider what happens if conservation laws are always valid and symmetries always maintained. Directly due to the fact that conservation laws can never be violated, the symmetry of the theory, together with a necessary generalization to five dimensions, leads, as we will see, to the very general consequence that backward and forward time evolution are both allowed as straightforward solutions of the dynamics, in that way producing possible consequences, which will be hypothesized in this paper. In doing so, we start with a discussion on the EPR paradox (initial point of the considerations involving Q.M. and Relativity); we carry on with considerations of conservation laws involving two arrows of time, which can give rise to the entanglement of physical systems; after that we generalize the approach to five dimensions in order to extend the time definition and to show how this leads to a scheme of unitary description of interactions; then we present a unifying description of Q.M. and Relativity, we match the results of our calculations with observational data and finally we present a picture of the possible last consequences on Time conception of such a theory.

2. Causality and Quantum Mechanics. The role of Conservation laws

2.1. EPR effect and Time Arrows

As it is well known, Einstein, Podolsky and Rosen criticized Quantum Mechanics, because, contradicting the locality principle, it leads to results which violates the causality principle [2]. In OQR, the complete redefinition of the entanglement concept, based on the impossibility of violation of conservation laws [16] and on the generalization to a five dimensional space, leads, instead to the recovery of the classical and fundamental idea for physics of Causality.

In fact, Q.M. requires that an interaction, even if only on a single part of a quantum system, determines a dependent evolution of the correlated quantities of the other parts, also when these parts are placed in "causally disconnected" regions of the space-time, (i.e. when they are so distant, that no direct interaction between the parts can occur, in the light speed limit). From the beginnings, Schroedinger tried to overcome the problem with the "qualitative concept" of entaglement, described as a sort of deep connection, not yet dynamically specified, able to rely two causally disconnected, but quantum related, objects.

The reasoning leading to this definition comes from a real necessity, because the entanglement, this concept unavoidable to explain why Quantum Mechanics works, would be otherwise simply impossible, without violating the causality principles and also the standard logic, which states that "it is impossible an interaction with an object without interacting with it in some way". Starting from the Schroedinger qualitative concept, later accepted and elaborated by several authors [17], let us rephrase here the statement (developed in a previous paper [18]) on causality, in this new Entanglement definition: "Two states, spatially separated and causally disconnected in four dimensional space that we ordinarily perceive, are entangled if an interaction with one of them can influence the other one, without in any way directly interact with it, because a four dimensional entanglement means that a causality nexus exists in a larger five dimensional physical space".

If we do not hypothesize another – and really physical - dimension, in which the two states are causally connected, – *so then restoring the causality principle-*, what we call entanglement would remain a "necessary but impossible" phenomenon. In our theory, the conservation laws are the first principle which determines all the following evolution, since the fact that they can never be violated leads to a mechanism to avoid such a violation, also in the cases in which, for the standard interpretations, the violation should occur. This mechanism is the topology change. It is the topology change which provides the dynamics allowing the very particular interaction, between two otherwise causally disconnected states, called entanglement, and so it is the topology change which makes possible the formation of a wormhole, connecting a black hole with its entangled white-hole [19], through a Goedel type change of topology [20].

The entanglement, in this picture, maintains the meaning of the underlying mechanism allowing interactions otherwise impossible, that Schroedinger initially gave to it, but with a dynamics providing an explanation of the phenomenon. However let us have a deep insight. The intrinsic characteristic of quantum mechanics is the existence of systems without any definite value of some measurable quantities, if one does not measure them. The significant point is that this feature comes out from the fact that the state of a system is a superposition of different states, and the only possibility left to the observation, is to interact in an irreversible way with the system, changing its state with the measure process. This description, mainly due to Bohr, is known as the Copenhagen Interpretation (CI), and in its framework, the irreversibility of the measure process, and consequently the irreversibility of the time direction, plays a key role. But the superposition of quantum states, that in CI is a characteristic of the microscopic description, may have macroscopic effects and this is at the base of the EPR paradox. Let us suppose (taking the Bohm's example [21]) to have a spin zero particle, or in general a bound state, which decays into two particles each of spin 1/2. As far as the spin of the particles in a definite direction is concerned, the state of the system is described by a state vector of the form:

$$|0\rangle = |\uparrow\rangle |\downarrow\rangle - |\downarrow\rangle |\uparrow\rangle \tag{1}$$

where we must recall that every single particle it is not a state by itself and the evolution of the state vector, with respect to the spatial distribution of the wave function, is not yet specified. There are not measurements at the times t such that $t_0 < t < t_m$ (where t_0 is the time of decay and t_m is the time of measurement) and therefore we do not know anything about the state of the system in that range of time. Now, at time t_m , we measure the spin σ_1 of the particle 1 and, even if the spin state of the system is still determined by (1), now the measure operation causes the collapse of the wave function in one of the two states $|\uparrow\rangle|\downarrow\rangle$ or $|\downarrow\rangle|\uparrow\rangle$.

However, the conservation of quantum numbers implies that, once the spin of particle 1 is measured and fixed, also the spin of particle 2 is instantaneously acquired, even if the particles are now far apart and not directly interacting. So then we have the following paradox: there are two particles without any direct interaction, but the state of the system is such that, if we fix the spin state of one particle, also the spin state of the other particle is instantaneously fixed. The non-local behavior (connected with the Bell inequalities) of this kind of systems has been tested by Aspect et al. [22] and the experimental results show that the non-locality is a true feature of Nature. Let us summarize the results of this experiment (and many others) using Sakurai's words (in a book revised by J. Bell himself [15]): "all the experiments made, have conclusively shown that Bell's inequalities (which are coming from the "locality prescription" of Einstein) are violated and violated in a way which is compatible, within the errors, with quantum mechanics predictions".

To explain this paradox, so crucial since at the intersection between Q.M and Relativity, we have developed a new approach, starting from the general remark that the Noether Theorem [23] states that for every conservation law of Nature a symmetry *must* exist. Starting from this consideration and *inside* the framework of conservation laws, it comes formally out the necessity of taking into account also a backward time evolution of the wave function, since dynamics, if derived from a variational principle, is always symmetric under time reversal transformations. This general consideration means that conservation laws intrinsically contain forward and backward causation, even if against common sense and local realism, which instead assume just one arrow of time. Below we show, again by general arguments, that Bianchi's identities, which are geometric identities directly connected to conservation laws, contain symmetric dynamics and, from such a dynamics, it is therefore possible to recover backward and forward evolution of the wave function. Starting our considerations from the statement that quantum matter can be described by a scalar field ϕ [24, 25], in a previous paper [26] it has been proposed a solution of the EPR paradox which does not formally change the frame of CI, so than all the features of CI are preserved, but the microscopic time flow, for the wave function, is different from the time flow of the observer. It must be underlined that this possibility, even if not intuitive and not referable to our macroscopic perception of reality, is really compatible with quantum mechanics because of our intrinsic ignorance of the state of the system in the time interval $t_0 < t < t_m$. More specifically, the dynamics of a scalar field ϕ , describing quantum matter on a (curved) space-time, is given by the stress energy tensor, which is a completely symmetric object and such a tensor has to satisfy the conservation laws, which are the contracted Bianchi identities. Sending to the quoted paper [26] for the full mathematical treatment, the final result is that the conservation of the contracted Bianchi identities implies the Klein-Gordon equation, providing the dynamics of ϕ , where it is important to stress that also the Klein-Gordon operator is symmetric. Now, being ϕ a scalar field, we want to remember that it can be interpreted as the product of two conjugate complex functions

$$\Phi = \Psi^* \Psi \tag{2}$$

and the Klein-Gordon equation can be written as:

$$(\Box + m^2)\Phi = (\partial_a\partial^a + m^2) (\Psi^*\Psi) = (\partial - im) (\partial + im) (\Psi^*\Psi) = 0$$
(3)

Eq. 3 can be split, for massive particles, in the cases:

$$(\partial - im)\Psi = 0 \quad (\partial + im)\Psi^* = 0 \tag{4}$$

$$(\partial - im)\Psi^* = 0 \quad (\partial + im)\Psi = 0 \tag{5}$$

where $(\partial - im)\psi = 0$, can be considered as a "forward" propagator, while $(\partial - im)\psi^* = 0$ is a "backward" propagator and vice versa for the other two.

Instead, for massless particles, the four conditions reduce to two

$$\partial \psi = 0$$
 , $\partial \psi^* = 0$ (6)

at the end, it comes out that a function (a superposition) of the form

$$\varphi(x) = \alpha_1 \psi(x) + \alpha_2 \psi^*(x) \tag{7}$$

where $a_{1,2}$ are arbitrary constants, is a general solution of the dynamics and the states ψ and ψ^* can be considered in our picture as *entangled*, since they can influence each other also when they appear disconnected. Summing up, the absolute validity of conservation laws gives rise to a symmetric dynamics (backward and forward evolution of the system) and the entanglement of states is naturally determined without *any arbitrary violation*. We want to stress that the four conditions (4-5) are satisfying Eq. (3), and this fact implies that backward and forward evolutions exist both for the ψ -field and the conjugate ψ^* -field. At the end, it seems that, in Minkowski space-time, all the folds of the light cone have the same dignity, even if we have been usually confined to investigate only the fold toward the future (the time arrow which we can normally perceive) and these considerations are completely general also in a curved space-time.

Finally, in order to save a logical connection on the observed fact that a measurement made on a particle seems able to affect the status of another, far apart, disconnected particle, it is important to notice that the statement can be re-expressed in terms of an effect of a measurement of a particle able to affect the system in the past: i.e. the (measured) relation of interference, known as the Bell inequalities, can be obtained if the particles decay in a state that "*depends on what will be measured*".

2.2. General conservation laws and equivalence principle, in a 5-D Space.

After the previous considerations, see also [26, 27, 28], it is now useful the explicit introduction of the *General Conservation Principle*: "the fundamental conservation laws are the only ones having an absolute meaning and keeping always their validity. This is the reason why, when it is otherwise impossible to keep their validity, they determine the entanglement phenomenon, which allows in any case their recovery, thanks to a topology change and a related change of time arrows. So, the conservation laws are always preserved, the topologies not" [16].

This principle, due to the fact that mathematical formalism does not prevent backward and forward evolution in time, allows an explanation in physical terms of non-local behavior for entangled systems, open a discussion on the *first principles* leading to a symmetric time and is coherent with the experimentally observed violation of the Bell inequalities and consequently of the Einstein locality principle [14, 26]. For the standard causality principle [14] and the Einstein-Podolsky-Rosen point of view [2], such a behavior is a paradox. Nevertheless, if we maintain conservation laws absolutely valid, entangled systems are naturally explained and the possibility to evolve backward to the past (i.e., in what we perceive as the past) is a general feature of nature, which emerge as soon as a conservation law has no other way to maintain its validity, even if, being confined in the forward fold of light-cone, we cannot directly experience it. As a consequence of this view, related to the concept of entanglement, there are topology changes taking place in order to preserve a conservation law, e.g., matter-energy in black hole dynamics, or quantum numbers in EPR effects [26]. In the first case, as we will see in the following, the entangled system is constituted by a black hole (BH) which dynamically evolves in a white hole (WH) (in fact a White Fountain [19]) through a topology change (the real essence of a worm hole) or, in other words, B.H. and W.H. are mould and body entangled together. The general feature of such a result is that the conservation laws can never be violated, and we want to outline that this is a general result, because of the connection among conservation laws, symmetries and first integrals of motion, has a deep physical meaning. On the contrary, topologies can change and so they can be considered as dynamical quantities. Moreover the hypothesized existence of backward and forward evolution leads to the necessity of a fifth dimension. Let us remember a very simple example, imagining an hypothetical onedimensional universe. In such a universe, a point placed to the left of another one could never invert its position exchanging the left with the right, while if we pass to a 2-dimensional universe it becomes obviously possible to exchange the relative position of the two. This is true also passing from four to five dimensions (making

possible the evolution in both directions of the time axis) [28, 29]. But this is not just a technicality, because we think in terms of 5-D universe with physical meaning, as we will show. In our approach, as the fourth dimension can be naturally related to time in general and special relativity, the fifth dimension can be naturally related to the masses of particles (so possibly giving rise to the differentiation of fundamental interactions [16, 30]). In other words, we can deal with a generalized five-dimensional mass-chronotope, every point of which is labeled by space, time and mass. It is relevant to note that we do not perceive the fourth time-dimension on the same foot of space-dimensions and the situation is analogue for this fifth mass-dimension. The set of conditions (4) and (5) are fundamental in our theory, since they give rise to forward and backward dynamics. From a relativistic point of view, in the cases (4) and (5) we are inside one of the folds of the light-cone, in the case (6) we are, instead, on the null surface and the role in distinguishing dynamics is played by the mass of particles [30]. The distinction of backward and forward dynamics depends on the dimension of the space-time we are dealing with, in a 5dimensional manifold, all particles are moving without preference of backward or forward evolution, while It is the embedding procedure from 5- to 4-dimensional space-time which gives rise to two dynamics (that is two arrows of time), but conservation laws are preserved in any case, also in the process of mass generation. Due to the fact that conservation laws are always and absolutely valid, also when, to keep such a validity, phenomena as topology changes and entanglement can emerge in 4D, we have a theory without singularities (like standard black holes) where unphysical spacetime regions are naturally avoided [26, 31] and this is a very significant characteristic of OQR. But there is more, because, thanks to this approach, we can formally deduce the equivalence principle. In fact the dimensional reduction can be considered from the geodesic structure and from the field equations points of view [32]. In the first case, starting from a 5D metric, it is possible to generate an Extra Force term in 4D, which is related to the rest masses of particles and then to the Equivalence Principle. In fact, masses can be dynamically generated by the fifth component of the 5D space and the relation between inertial mass and gravitational mass is not an assumed principle, as in standard physics [27], but the result of the dynamical process of embedding. It has to be stressed that an "amount of work" is necessary to give the mass to a particle. An effective mass is recovered also by splitting the field equations in a (4+1) formalism, the fifth component of the metric can be interpreted as a scalar field and the embedding as the process by which the mass of particles emerges. It is very important the fact that particles acquire the mass from the embedding of geodesics and from the embedding of field equations, because this is the reason why the chronological and geodesic structures of the 4D spacetime are the same: they can be both achieved from the same 5D metric structure, which is also the solution of the 5D field equations. By taking into account such a result in 4D, the result itself naturally leads to understand why the metric approach of General Relativity, based on Levi-Civita

connections, succeeds in the description of spacetime dynamics, even without resorting to a more general scheme, as the Palatini-affine approach, where connection and metric have to be, in principle, considered distinct. The reduction process leads also to a wide class of time solutions, including two-time arrows and closed time-like paths, so again recovering the concept of causality, thanks to the necessary introduction of backward and forward causation [1]. We will see that Open Quantum Relativity allows to face Quantum Mechanics and Relativity under the same dynamical standard (a covariant symplectic structure [33]) and this occurrence leads to frame several paradoxes of modern physics under the same dynamical scheme. But we should stress here that the only way to perceive a backward evolution, is under the extreme conditions in which a conservation law, being otherwise violated, determines a space–time breaking via a topology change and this phenomenon is allowed by the existence of a fifth dimension of real physical meaning

2.3. Contemporaneousness and light speed

In Open Quantum Relativity, another important question, coming always from the EPR effect, finds a possible solution. The question of the supposed contemporaneousness of the effects of an interaction on a part of a system, with the induced effect on the other part (placed far and non directly interacting, but entangled), question becomes even more important after experimental results claiming for instantaneous quantum teleportation [12, 13]. The long series of Bob & Alice (receiver & emitter) papers is the most known example of it [12] and a special relevance deserves the very important claim by Bouwmeester et al. [13], where, following Bennett et al. suggestion [34]: "It is possible to transfer the quantum state of a particle into another particle, provided one does not get any information about the state in the course of the transformation". Bouwmeester presented experimental evidences of an effect of polarization, given on one of a pair of entangled photons, transferred on another one, without any direct standard interaction between them. The experiment is, at the end, an "entanglement swapping", from a photon of an entangled pair to a third one and, at the end, photon 1 is no longer available in the original state, but photon 3 is now in that state as a result of instantaneous teleportation.

It remains the fact that it is necessary to send to the receiver, in standard way, the information of the state of the entanglement (one possibility among four) between photon 1 and photon 2, and this fact opens a big discussion on the real meaning of teleportation. Anyway, besides the possible and very important applications, these experimental results are suggesting some comments:

i) it seems possible to determine the entanglement also between particles (and probably also between more complicated objects) without the same origin, so opening the doors to a conceivable general technique; ii) it seems possible to send information, via entanglement, without any destroying influence of the environment and as far as one wants; iii) it seems possible to send information instantaneously, even if a conventional message is necessary to inform and check. The last two points seem to indicate a $\Delta t = 0$ in the transfer operation, the only way to send information instantaneously and without any environmental influence. This really seems in contradiction with Special Relativity, even if there are authors that are saying that, due to the necessity to send also a conventional message, this would restrict again the phenomenon in the limits of Relativity, because it could not be used to send information faster than light. This reasoning, however, is not fully convincing, because, despite our capability to utilize the information only in conventional way, it remains the fact that the phenomenon would be instantaneous. This is a problem, because questioning the relativity, but a problem only if continued to be treated in terms of super-luminality, whereas is no longer a puzzle if treated in terms of *A-Luminality*.

Let us take into account (fig. 1) a defined and bounded surface: it is evident that it would take time to go from a point to another one, and even from a point at the upper side to the corresponding image point at the lower side of such a surface, but this time is reduced to zero, if a mechanism exists to make a "hole" and get directly, once defined an orientation, the bottom from the top.



It is straightforward to see, once defined the transformation for time and space intervals $\Delta t'$ and $\Delta x'$:

$$\Delta t' = \Delta t (1 - v^2/c^2)^{1/2}, \quad \Delta x' = \Delta x (1 - v^2/c^2)^{1/2}$$
(8)

— 136 —

that the reduction to zero of the space interval $\Delta x'$ implies, in the second of eq. 8, that $v \rightarrow c$, so than $\Delta t' \rightarrow 0$ for the time interval $\Delta t'$.

The situation is that one illustrated in fig.1, the travel is from A to B (or viceversa), the mechanism is the change of topology and the deep reason of this fact is the physical necessity to save the conservation laws, without overcoming the light speed. This kind of travel is an *A-luminal* travel.

In our theory, the conservation laws are the first principle which determines the following evolution, including the phenomena in which this would be apparently impossible, through the topology change which makes possible, for example, the "hole" connecting the two points of the above figure. But the fundamental concept that has to be stressed here, is the "A-Luminality", because it saves contemporarily conservation laws and causality principle, and it seems moreover the onlyone compatible with the necessity to extend, but not contradict, the Relativity.

2.4. A covariant symplectic structure for Quantum Mechanics and Relativity

A self consistent quantum field theory of spacetime has been searched for long time, but, using standard approaches, without success. In Open Quantum Relativity [1], in order to formulate the General Conservation Principle at fundamental (i.e. quantized) level, it is necessary to search for a covariant symplectic structure, starting from invariant quantities. It is well known that all the attempt to quantize general relativity, till now, get only non-resolutive or contradictory results, so that a quantum theory of gravitation does not exist yet in a closed and self-consistent form. For instance, in the Arnowitz, Deser and Misner (ADM) formulation of the gravitational field, the authors get an Hamiltonian definition of the field, but with a reduction (3+1) of the dynamics, where Time is a priori distinct from the spatial degrees of freedom. In that way the field equations con be considered as Hamilton equations, but, nevertheless, the more fundamental and original characteristic of General Relativity disappears: the general covariance. In this sense a canonical formulation of gravitational field is not covariant. Another contradiction of the standard approaches, is that it is not possible to define a quantum field theory in which the dynamical degrees of freedom, the metric tensor gnv and the background coincide, as instead it is for gravitational field. Starting from these considerations, it is necessary to search for invariant structures, anyhow conserved, capable of giving rise to a symplectic structure, independently of metric tensor and allowing to write covariant Hamilton equations. In this way the General Conservation Principle can be formulated at fundamental level, so leading, in principle, to a general quantization scheme. In this perspective, it is possible to build Hamiltonian invariants starting from vectors, bivectors and in general tensors, covariant or contravariant, appropriately contracted. These quantities are intrinsically conserved and, considering the covariant derivatives of the component vectors, it is possible to obtain a covariant symplectic structure which is naturally equipped of Poisson brackets and

so Hamilton equations. The principal ingredients of this approach are the Hamiltonian invariants appropriately built and the affine connections (not the metric), which are giving rise, through the covariant derivation, to the Hamilton equations.

We must underline that the consideration of the affine connections, instead of the metric, to describe a common structure for booth G.R. and Q.M is a fundamental distinction point of OQR, and, moreover, this covariant symplectic scheme is completely general [1, 33]. Through the specification of the vectors (or tensors) which constitute the Hamiltonian invariant, it is possible to get any specific field theory, because the scheme is generally valid and can be applied to any field theory.

For example, if the vectors are 4-velocities on curved space-times, the two Hamilton equations correspond to the equation of geodetics and to that of geodesic deviation (the equations for q_i and p_i of the usual Hamiltonian mechanics), while if the vectors are 4-potentials of the electromagnetic field, we obtain the Lorentz Gauge and the Maxwell equations. At the end, beside what is of direct interest of this work, there is a covariant symplectic structure allowing an unified view of interactions, thanks to the conservation laws [33].

3. The role of Time in Modern Physics

3.1. Toward a new Time definition in the Framework of Relativistic Theories.

We know that General Relativity introduced the concept of relative time, so suggesting the first idea of Time Travel. In fact, already in standard Relativity it is conceivable the possibility of a relativistic time travel in a "Relative Future" (relative, because nobody can imagine yet to travel in his own future, but only, if accelerated for a period in a rockets, in the future of "the others" on Earth, a future, moreover, perturbed by his absence during that period). But it is in the relativistic theories that the time travel hypothesis takes a more general meaning, especially if such theories are taking into account also quantum mechanical effects. If we fix our attention on the possibility (by conservation laws induced) of change of topology, we can get, as an example, a Goedel type condition of a closed time-like curve (CTC), recovering its deepest meaning of opening the doors to travel even back in time. In fact a closed time line, which Goedel [20] first presented as a purely formal possibility, in an unconventional solution of Einstein equations, (simplifying, a cylindrical coordinates choice, where the longitudinal-one is spatial and the circular is temporal, instead of the standard contrary) it makes possible track back in time the footing done, so then "to pass again" in the past. But, what for Goedel was a mathematical conjecture, here becomes a physical hypothesis, because based on a dynamics (the forward and backward evolution of the split Klein-Gordon equation), on a "necessity" (to save the conservation laws and the causality principle), on a conceivable footing (the induced topology change defined in an appropriate mathematical domain) and finally, more in general, on a coherent theory able to propose an explanation of several open problems and reproduce several experimental observations.

After Goedel, many authors showed that, far to be an exception, the possible existence of natural CTCs in the Universe should be an infinity, unless the existence of hypotetical rules (like, for instance, the Hawking Chronology Protection) forbidding it. Nevertheless, up to now, the proposed forbidding rules are based on semi-classical considerations, so then no longer holding in the framework of quantum gravity, because they are an un-appropriate combination of quantum matter field with a classical spacetime device [35]. Let us now enter in some general properties of spacetimes with CTCs [36, 37, 38, 39], remembering that solutions of Einstein's equations allowing CTCs have been known since a long time. The earliest example of such a spacetime is a solution obtained by Van Stockum in 1937 [40], which describes an infinitely long cylinder of rigidly and rapidly rotating dust, CTCs are also present in the interior of Kerr black hole in the vicinity of its ring singularity, and other examples of spacetimes with CTCs are in [41]. In general, a spacetime can be divided into *chronal regions*, without CTCs, and *achronal regions* which contain CTCs. The boundaries between the chronal and achronal regions are formed by chronology horizons, precisely chronal regions end and achronal region begins at a *future chronology horizon*, while achronal regions end and chronal region begins at past chronology horizon. Thus, a-chronal regions are intersections of the regions bounded by both of these horizons.

In the framework of General Relativity, in order to build a time machine (which, in this context, means in general a region with CTCs) by using a wormhole, one needs to assume that there exists the possibility, in principle, to make it long living and traversable, which, in standard approach, needs the violation of the averaged null energy condition [42]. But this is in standard view, because the whole time conception, only apparently intuitive, is perhaps so dramatically changing in the framework of post relativistic theories, that the our whole knowledge of physics will be deeply modified. For instance, in OQR, we will see that the Many Worlds interpretation of Quantum Mechanics [43] is not a useful, but a necessary ingredient of the approach. Before going deeper in OQR consequences, verifying the theory output with experimental observation, let us remember the open problems which can get a possible explanation from this theory. Namely: the explanation of the EPR phenomenon avoiding the conflict between Relativity and Quantum Mechanics [26]; the generalization of entanglement concept as a gate, through topology changes, instead of a paradox [18]; the new Black-Hole definition overcoming the singularity problem together with tha Gamma Rays Burst explanation [29]; the dynamical deduction of the equivalence principle [32]; a dynamical scheme for the unification of the different interactions [30]; and finally a description, through a common symplectic structure, of both Quantum Mechanics and General Relativity [1, 33].

We should stress that the whole question of time, and of the time machine

hypothesis, is completely changed by this new theory, because "moving in time" is no longer an extreme possibility only due to a relative motion of two frames on the same time arrow, but a more general possibility due to the existence of a second and backward directed time arrow, so "backward in time" means that, in the new theory, there is also another very general possibility and the real fundamental question is how to get "physically" a backward time arrow, that we cannot ordinarily perceive, but which must be existent [1,18,26,28]. In order to substantiate the previous statement, let us recall the equations describing quantum matter:

$$\psi(x) = e^{-ikx}u(k) \quad , \quad \psi^*(x) = e^{ikx}u^*(k) \tag{9}$$

Wich can be interpreted, respectively, as progressive and regressive solutions in four combination. It comes out that a function (a superposition) of the form:

$$\varphi(x) = \alpha_1 \psi(x) + \alpha_2 \psi^*(x) \tag{10}$$

Where $\alpha_{1,2}$ are arbitrary constants, is a general solution of the dynamics and the states ψ and ψ^* can influence each other, also when they are, in standard physics, considered disconnected. As we will see, the resulting scheme shows itself properly working in explaining many paradoxes and shortcomings of modern physics, with the minimal necessary number of parameters in comparison to all existing theories (like Strings, or Supergravity). In the context we are discussing here, thanks to conservation principle avoiding spontaneous symmetry breakings, particularly interesting is the fact that entangled gravitational systems, constituted by black holes, wormholes and white holes, emerging through topology changes, can be stable, so that time travels and time machine becomes, theoretically, a conceivable possibility [19, ...] (and, moreover, starting from 5D dynamics, CTCs are ordinary solutions of field equations). It is evident the full recovering of General and Special Relativity (because we do not need to travel faster than light), but, nevertheless, the mechanism of relative motion between two frameworks, which is generating the hypothesis of time travels, is clearly not the same if applied on one or two time arrows. In this sense, we deal with "Open" Quantum Relativity.

3.2. OQR vs experiments and observations

The O.Q.R. is a general theory and the resulting scheme is self-consistent and able to solve several problems of contemporary physics, far beyond the starting point of the EPR paradox or the very recent deduction of the equivalence principle, because several experimental confirmation comes out from cosmology and astrophysics.

For instance the search, starting from general laws, of the initial conditions [44] of the Universe evolution, in our scheme leads to the relevant result that the

existence of conservation laws (i.e. symmetries of the system of the cosmological equations) dynamically determines that the so called "wave function of the Universe" selects first integrals of motion, whose cosmological solutions are "observable universes" (in the same sense of space-times classically considered [45]), so proposing a solution for an old problem [46, 47]. The OQR approach is confirmed in several cosmological and astrophysical observations, because the model coming out from the reduction process from 5 to 4 dimensions, seems largely fulfill the recent observational results. Today, combining data from Supernovae of Ia type, from cosmic background radiation (especially COBE, BOOMERANG and WMAP), from galactic clouds Lya and from the large scale galaxies clusters distribution [48], it comes out a spatially flat cosmological model, whose dynamics is dominated for 30% by matter (baryonic and dark) and for 70% by dark energy (which can be considered either cosmological constant, or the so called Quintessence scalar field, or same other unidentified dynamical entity). The characteristic of this observational frame, is that the Universe seems to accelerate, instead of the contrary as in Standard Cosmological Model predictions, and moreover the possible sources of dynamics remain totally unknown. With our mechanism, instead, we succeed in reproducing the cosmological parameters in agreement with the observed values, and this starting from a general theory without conditions arbitrarily introduced. In particular, taking into account our cosmological model [19] derived from the embedding process from 5 to 4 dimensions, we obtain $\Omega_M \approx 0.3$ (density parameter of baryonic and dark matter), $\Omega_{\Lambda}\approx~0.7$ (density parameter of dark energy), $q_0 \approx$ - 0.5 (deceleration parameter), $t_0 \approx$ 14 Gyrs (estimated age of the Universe), $H_0 \approx 65$ Km / sec / Mpc (Hubble parameter) so reproducing correctly, and at the same time, the observed values of the five more significant cosmological parameters. We have to stress that also the scalar field, whose dynamics is ruled by Klein-Gordon equation (i.e., as we have seen, by the Bianchi identities) is recovered in our frame. Another astrophysical problem which can be settled in this scheme, is that of dark matter that should be supposed to explain the observed dynamics of the rotation curves of the galaxies. In fact the effective theory of gravitation, that emerges from the process of dimensional embedding, implies a variable gravitational coupling, in which the Newton constant results to be a parameter dependent from the scale [46]. In other words, the Newtonian potential, able to describe the gravitational effects up to Solar System scale, must be corrected by Yukawa-type terms, depending by characteristic lengths, like, for instance, the typical scale of galaxies (from 10 to 100 Kpc).

In this scheme, the enormous quantities of dark matter necessary to the astronomers to explain the observed dynamics of the galaxies (up to 90% of the total mass of a typical galaxy) are no longer necessary, because the scale dependence of the gravitational interaction can provide a correct frame, without the addition of elements still unidentified.

But also other astrophysical problems can find together a possible solution in

the scheme of the Open Quantum Relativity like Black Holes and Gamma Rays Bursts [31]. To conserve in a B.H. the total mass-energy, we consider an entangled system in which the B.H. dynamically evolves, through a topology change (the true essence of a Worm hole), in a White Fountain, that we can observe as a Gamma Rays burst. In fact, those extremely energetic bursts of Gamma Rays was, up to now, without a well established mechanism able to explain their origin, power, polarization and spatial distribution, but, in the light of the OQR, they can be considered as the signature of dynamical process of entangled astrophysical systems, Black Hole - Worm Hole - White Hole. The idea is that the absolute conservation of the Mass-Energy, during the gravitational collapse (Black Hole), induces a topology change (Worm Hole) in such a way that, in another space-time zone, it emerges an energy fountain (White Hole). This process can induce an explosive emission (peak of the Gamma Rays Burst) with a following tail (spectrum of Gamma Rays Burst). From the calculation it emerges that if the Black Hole is generated by a star big enough (for example of the order of 10 solar masses) it is possible to obtain the energetic values of Gamma Rays Bursts. Moreover, the GRB distribution seems like the distribution of the evolved stars (i.e. stars able to generate Black Holes) and finally the observed polarizations can be naturally explained thanks to the passage of the energetic flux through the Worm Hole [31]. Finally a recent result is the correct reproduction of the observational limits of neutrino oscillations, derived from induced gravitational field in our theoretical scheme [49]. As a conclusion of this short review of experimental evidences, it seems that several open problems of the contemporary physics, can be coherently explained (and without elements arbitrarily introduced) thanks to the General Conservation Principle in the frame of the Open Quantum Relativity.

4) OPEN QUANTUM RELATIVITY AND TIME

Before entering in the possible last consequences of Time conception in OQR, let us recall the main characteristics of this theory. At the end, the Open Quantum Relativity [1] is a coherent dynamical theory, capable of including in its scheme new phenomena very difficult to be included in the standard descriptions, and based on a General Conservation Principle together with a symplectic description for both Quantum Mechanics and General Relativity. The scheme is the following:

a) We consider that the conservation laws are always valid, i.e. they can never be violated [16], so then symmetries are always conserved.

b) The framework, in which the new approach takes place, is a five dimensional physical space [16, 29].

c) A correlation between two quantum objects causally disconnected (i.e. so far each other that they cannot interact in the limit of the light speed) means that they are entangled, i.e. causally connected in a space scaled up to 5 dimensions

[16,29]. This means, in the standard four dimensional space-time, that the causality can be generally recovered, admitting that the Present can be influenced also by the Future and that the contemporaneousness of interaction effects on entangled objects is an A-Luminal process.

d) In five dimension, backward and forward time evolution are both allowed and without distinction. While, in our 4D space-time, it emerges the distinction of two time arrows, together with our limitation in perceiving only one of them, eccept for the effects of an entaglement condition, through a topology change [16, 29].

e) The possibility to describe the Relativity in quantum form, is based on the identification of a symplectic structure common to both Relativity and Quantum Mechanics, so making possible to express relations trough Poisson brackets of affine connections (instead of the attempt of quantization of the metric) [1, 33].

After the description of the features of the Open Quantum Relativity, and reminding to the quoted papers for the full treatment, let us now stress the results on the specific topic of this paper: the long term possible consequences on time conception. It is clear that, once derived the natural existence of a second time arrow backward oriented, the whole definition of a time machine is changing, because, in this case, a CTC will be not a exception coming out only from very particular conditions, but a possibility coming from general feature of the Nature and the problem is re-oriented to the search for the opportune gate to get it. What we mean is that: while the concept of time travels in General Relativity is only related to the difference in spacetime among different frames, here, in the new approach, the relative difference can be directly taken into account on two time arrows and this means that a closed timelike path does not need, to exist, a topology change imposed "by hand". The new theory, even not entering in any detailed hypotesis for the technical devices of an hypothetical Time machine, provides nevertheless some principles on which such a machine should be, at least, based, because it is fixing the limits in which such phenomenon will be eventually possible. In fact, only in a situation of impossibility to avoid a violation of a conservation law, it happens that the nature reacts changing the topology, so then we should, first of all, find in the universe such a situation or be able to recreate it. In the first case (up today knowledge), the "laboratory" for such an experiment should be a black hole, in the second case, an "entanglement machine". This would be a kind of machine very difficult to design yet, but that we cannot, in principle, totally exclude [50].

In fact, there are works of authors [51] which demonstrate that the entanglement of two macroscopic systems, does not need the entanglement of every component of them in correspondence one-to-one (which will be probably impossible to obtain) but only a correspondence at least under the limit of the uncertainty principle (certainly not easy, but, in principle, not impossible). Under the realistic hypothesis that the travelers want to come back (and stay alive), further conditions are then necessary: the hypothetical spacetime craft has to move together with the voyager, because the existence of two time arrows can make any time travel independent, without an automatic "return ticket" and, moreover, the approach to a time tunnel should be A-Luminal to avoid that the travelers could be squeezed in elementary particles (one can postulate a null pressure in a wormhole, but, before getting it, in the black hole proximity the gravity pressure would destroy the structure of any approaching object). Furthermore, in OQR frame, the Many Worlds Theory should be taken as a necessity, in order to avoid new and deeper paradoxes induced by backward time travels, like the famous hypothesis of a time traveler killing his grandmother when she was young (before his own father birth) so destroying himself and making impossible to kill grandmother. Only the Many Worlds Theory, in its full version [43] of an infinity of universes (each one representing a virtual possibility of evolution that can become a reality), can settle this puzzle, saving Time travel general hypothesis, causality, and logic together. In this picture, every standard physical universe can be considered a "local fact". It is very difficult, today, to estimate all the consequences of the above considerations and mainly those concerning a "time machine", nevertheless one should derive some conclusions to open an useful debate.

5. OPEN TIME. UP TO THE LAST CONSEQUENCES.

Nobody can say, today, if a time machine will be really realized in the future, but, if possible, it has to obey, at least, to the following criteria from the above principle [see for the full treatment [54, 55].

1. It has to be possible to reach critical conditions of violation of a conservation law, in such an otherwise unavoidable way, to give rise (to evade it) to the topology change phenomenon, which is at the basis of spacetime tunneling possibility.

2. It has to be possible the entanglement and the entanglement swapping of spacetime bounded regions, which can be entangled "as a whole", in the sense that the fraction of their components, not mutually entangled one-to-one, has to be lower of the limit of uncertainty principle [51].

3. It has to be possible that what is going to be involved in a spacetime tunneling (from a simple information up to a large spacetime craft) can be considered only a perturbation of the large scale space-time, again in the limit of uncertainty principle[51]

4. It has to be possible the existence or the emergence of CTCs, and moreover to be in the framework of Many Worlds Theory [43,53]. In fact, even the fact of traveling back in time, can generate a cascade of cumulative effects, such that to exceed in the following the uncertainty limits, in that way violating, in our 4-D Universe, the causality law, the recovery of which can be assured only by the existence, in 5-D, of several 4-D universes, as many as the possibilities of changing.

5. It has to be possible to estimate the permanence of entanglement conditions between two space-time region "tunneled" together, but following two independ-

ent world line, i.e. to estimate when the cascade effects of perturbations change enough their evolution, to exceed the uncertainty limit, so disconnecting the two spacetime regions and giving rise to two distinct 4D Universes.

6. It has to be possible to calculate exactly the spacetime region where and when the travel is going to end (i.e. well posed Cauchy boundary conditions).

7. It has to be possible to know how and when realize, following the conditions, the inverse operation to come back home. In any case it seems necessary that the technical device (whatever it will be) that initially produces the requested conservation law violation, should travel with the passenger, because the inverse operation is not generally automatic, but should obey to the same laws as the initial one.

8. It has to be possible a topology change not induced by destructive mechanism (i.e. not undergoing through a process of separation and recombination, perhaps possible, but eventually incompatible with the life conservation) but a mechanism able instead to give rise to the transformation of the system as a whole.

9. It has to be found topology change induced by mechanisms based on physical phenomena compatibles with life and particularly it has to be possible to obtain an A-luminal mechanism to get the time-tunneling.

10. (Last but not least). It has be possible to conceive technically a *gedanken* spacetime craft (and then a real one) in order to use natural time-tunnels and/or a *gedanken* time machine to realize artificial time-tunnels.

The relative time is not ceasing to change the physics, starting from the initial definition in Relativity up to now, and it seems indeed that its revolutionary effects, coming as a cascade from the first revolution, are able of producing even more consequences in the next future. Also the conception of time machine, despite the difficulties that we have even only in conceiving it, comes out from the actuality of a problem, the "relative time" deep meaning, which is real and far to be satisfactory solved.

We are in the middle of a crossing, we should accomplish this transition and find a new synthesis of the scientific discoveries of last century, which are tesserae of a mosaic still incomplete. In any case, we do not believe that any semi-classical approximation will be enough to settle this modern puzzle. Like many other people, we considered the necessity of a Quantum Relativity as an issue which was not possible to postpone any longer, as a tool necessary for solution of modern paradoxes and shortcomings, directly coming out from first principles of physics. Now, due to the fact that there are experimental evidences not only for the well established standard effects (e.g. the lengthening of the life of accelerated particles) but also for new ones (like the instantaneous teleportation via entanglement effects) it seems impossible to simply close the doors to any time tunneling hypothesis and so the necessity for a new theoretical framework, to include also this extreme possibility, seems to us unavoidable. A General Conservation Principle, together with two time arrows; the generalized topology change concept in a Many Worlds framework; a fifth dimension to "open" the Relativity and to provide the settlement in which the Many Worlds interpretation can take place; a set of equations capable of giving dynamics of many different new phenomena; are all parts of an approach which seems to us necessary to understand what happens in today physics.

In this paper, we presented a points of view on a matter which is still largely questionable, because not yet definitely settled. On this topic, all scientific consideration has to be seriously taken into account, but the difficulties, in this work, lay on the fact that the scientific community should try to solve the existing paradoxes without introducing theories involving too many free parameters and dimensions (eventually introducing even more paradoxes and shortcomings).

The OQR theory, here recalled to foresee the possible last consequences on Time, is an effort in this direction.

REFERENCES

- [1] G. Basini, S. Capozziello, General Relativity & Gravitation, 37, (2005) 115.
- [2] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47 (1935) 777.
- [3] E.W. Kolb, M.S. Turner, The Early Universe, Addison-Wesley, Redwood, 1990.
- [4] S.O. Alexeyev, M.V. Sazhin, Gen. Relativ. Gravit. 30 (1998), 1187.
- [5] S. Alexeyev, A. Barrau, G. Boudoul, M. Sazhin, Cl.Q. Gravity 19 (2002) 4431.
- [6] J.D. Bekenstein, Black Hole Thermodynamics, Physics Today, 1980.
- [7] C. Quigg, Gauge Theories of Strong, Weak, and Electromagnetic Interactions, Addison– Wesley, Reading, MA, 1983; P.D.B. Collins, A.D. Martin, E.I. Squires, Particle Physics and Cosmology, Wiley, New York, 1991.
- [8] G. Basini, A. Morselli, M. Ricci, Riv. Nuovo Cimento 12 (1989) 4.
- [9] A. Zichichi, Riv. Nuovo Cimento 24 (2001) 12.
- [10] A.D. Dolgov, M.V. Sazhin, Ya.B. Zeldovich, Basic of Modern Cosmology, Editions Frontieres, Gif-sur-Yvette, 1990.
- [11] V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk 166 (1996) 493.
- [12] C.H. Bennett et al. *Phys. Rev. Lett.* **70**, 1985 (1993).
 M. Zukowski et al., *Phys. Rev. Lett.* **71**, 4287 (1993).
 H. Weinfurter, *Europhys. Lett.* **25**, 559 (1994).
- [13] D. Bouwmeester et al. Nature 390, 575 (1997).
- [14] F. Selleri (Ed.), Conceptual Foundations of Quantum Mechanics, Benjamin, Menlo Park, CA, 1988; F. Selleri (Ed.), Quantum Mechanics versus Local Realism, Plenum, London, 1988; J. Bell, Rev. Mod. Phys. 38 (1966) 447; J. Bell, Physics 1 (1965) 195.
- [15] J.J. Sakurai, Modern Quantum Mechanics, Revised Edition, Addison-Wesley Publ. Co., New York, 1994. 10.
- [16] G. Basini, S. Capozziello, G. Longo, Phys. Lett. 311 A, 465, (2003).
- [17] C. Brukner, M. Zukowski, A. Zeilinger, Quantum-ph/0106119 (2001).
- [18] G. Basini S. Capozziello, Europhys. Letters 63, 166 (2003).
- [19] G. Basini, S. Capozziello, Astroparticle Phys 21 (2004), 543
- [20] K. Goedel, Rev. Mod. Phys. 21, 447 (1949).
- [21] D. Bohm, Quantum Mechanics, Prentice-Hall, Englewood Cliffs, N.J. (1951).
- [22] A. Aspect, P. Grangier, G. Roger, *Phys. Rev. Lett.* 47, 460 (1981). A. Aspect, P. Grangier, G. Roger, *Phys. Rev. Lett.* 49, 91 (1982). A. Aspect, J. Dalibard, G. Roger, *Phys. Rev. Lett.* 49, 1804 (1982).

- [23] C. Itzykson, J.B. Zuber, Quantum Field Theory, McGraw-Hill, Singapore, 1980.
- [24] M. Kaku, Quantum Field Theory, Oxford Univ. Press, Oxford, 1993.
- [25] N. Birrell, P.C. Davies, Quantum Fields in Curved Space, Cambridge Univ. 1984.
- [26] G. Basini, S. Capozziello, G. Longo, Gen. Relativ. & Gravit. 35 (2003) 189.
- [27] E. Scrhoedinger, Space-Time structure, Cambridge Univ. Press, 1960, Cambridge.
- [28] G. Basini, S. Capozziello, Rivista Nuovo Cimento N 12 (2004).
- [29] G. Basini S. Capozziello, Europhys. Letters 63, 635 (2003).
- [30] G. Basini, S. Capozziello, Gen. Relativ. & Gravit. 35, 2217, 2003.
- [31] G. Basini, S. Capozziello, G. Longo Astroparticle Phys. 20, 457, (2003).
- [32] G. Basini, S. Capozziello, Progress in Physics, Vol. 3, 36, July 2007.
- [33] G. Basini, S. Capozziello, Modern Physics Letters A, vol. 20, 4, 251, (2005).
- [34] C.H. Bennett, Phys. Rev. Lett. 70, 1895 (1993).
- [35] V.P. Frolov and I.D. Novikov, Phys. Rev. D42, 1057 (1990).
- [36] M. Visser Lorentzian Wormholes: From Einstein to Hawking, American Institute of Physics Press (1995).
- [37] K. S. Thorne, in GRG13: General Relativity and Gravitation 1992 Proceedings of the 13th International Conference on General Relativity and Gravitation, Cordoba, Argentina, 1992, Bristol Institute of Physics, p. 295.
- [38] J.L. Friedman et al. Phys. Rev. D42, 1915 (1990).
- [39] S.W. Hawking Phys. Rev. D46 2 (1992).
- [40] W.J.Van Stockum, Proc. Roy. Soc. Edin., 57, 135 (1937).
- [41] F. de Felice, Nuovo Cim. 65, 224 (1981).
- [42] S.W. Hawking, G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Univ. Press Cambridge 1973.
- [43] J.J. Halliwell, Nucl. Phys. B266, 228 (1986).
 - J.J. Halliwell, Phys. Rev. D36, 3626 (1987).
 - J.J. Halliwell, in *Quantum Cosmology and Baby Universes*, Eds. S. Coleman S., Hartle. J.B., Piran T., Weinberg S., World Scientific, Singapore, (1991).
- [44] J.B. Hartle and S.W. Hawking, *Phys. Rev.* D28, 2960 (1983). De Witt B. and Stora R., North Holland, Amsterdam (1984).
- [45] M.S. Morris, K.S. Thorne, and U. Yurtsever, Phys. Rev. Lett. 61, 1446 (1988).
- [46] G. Basini, S. Capozziello, Int. Journ. Mod. Phys. D 13, 1129, (2004).
- [47] G. Basini, S. Capozziello, F. Bongiorno, Int. J. Mod. Phys. 13, 717, (2004).
- [48] G. Basini, S. Capozziello, M. Ricci, F. Bongiorno, Int. Journ. Mod. Phys. D13 (2004).
- [49] G. Basini, S. Capozziello, General Relativity & Gravitation, V. 38, March 2006.
- [50] V.P. Frolov, I.D. Novikov, Phys. Rev. D 42, (1990) 1057.
- [51] B. Julsgaard, A. Kozhekin, and E.S. Polzik, Nature 413, 400 (2001).
- [52] G. Basini, S. Capozziello, Int. J. Mod. Phys. D, vol. 4, 583, (2006)
- [53] G. Basini, S. Capozziello, Progress in Physics, Vol. 4, 65, 2006.
- [54] G. Basini. S. Capozziello, *Classical and Quantum Gravity Research*, Ed. M.N. Christiansen and T.K. Rasmussen, Nova Science Pub. New York 2008.
- [55] G. Basini. S. Capozziello, "Lie groups: New Research.", Ed. F. Columbus, Nova Science Pub. New York 2009.