# A Cusped Prismatic Shell-like Body With the Angular Projection under the Action of a Concentrated Force (**) 

Abstract. - The elastic equilibrium problem for a cusped (tapered) prismatic shell-like body with the angular projection under the action of a concentrated force is solved in the explicit form within the framework of the zero approximation of I.Vekua's hierarchical models of prismatic shells. The thickness of the prismatic shell-like body is proportional to the angle bisectrix coordinate raised to a non-negative exponent. When the angle and exponent equal to $\pi$ and zero, respectively, the above solution coincides with the well-known solution of the classical Flamant problem [1].

## 1. - Introduction

In fifties of the XX century, I. Vekua suggested a new mathematical model of elastic prismatic shells (i.e., of plates of variable thickness in case of symmetric shells) which was based on the expansion of fields of displacement vectors, strain and stress tensors of the three-dimensional theory of linear elasticity into orthogonal Fourier-Legendre series with respect to the variable of the prismatic shell thickness. Considering only the first $N+1$ terms of the expansions, he obtained the $N$-th approximation. Each of the approximations $N=0,1, \ldots$ can be considered as an independent mathematical model of prismatic shells from the above chain of the hierarchical models, e.g., in case of symmetric prismatic shells (i.e., plates) the approximation $N=1$ actually coincides with the classical plate bending theory. In sixties, I. Vekua offered the analogous mathematical model for thin shallow shells. All his results concerning plates and shells are collected in his monograph [2]. At the same time he recommended to investigate cusped prismatic shells, i.e., prismatic shells whose thickness vanishes on a part of the plate projection boundary or on the whole one (about investigations in this direction see survey [3], [4], and also I. Vekua's comments in [2], p. 86).
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The present paper deals with the Flamant type problem for a cusped prismatic shelllike body in the approximation $N=0$

## 2. - Cusped (TAPERED) PRISMATIC SHELLS

Let $O x_{1} x_{2} x_{3}$ be a Cartesian coordinate system. Let us consider elastic body (s.c., prismatic shell) which is bounded from top and from below by the surfaces $x_{3}=\stackrel{(+)}{b}\left(x_{1}, x_{2}\right)$ and $x_{3}=\stackrel{(-)}{b}\left(x_{1}, x_{2}\right),\left(x_{1}, x_{2}\right) \in \omega$, respectively $(\omega$ is a projection of the body in the plane $x_{3}=0$ ), and (from lateral side) by a cylindrical surface parallel to $O x_{3}$. The difference

$$
2 h=\stackrel{(+)}{b}\left(x_{1}, x_{2}\right)-\stackrel{(-)}{b}\left(x_{1}, x_{2}\right) \geq 0
$$

will be called a thickness of the above body. The boundary $\partial \omega$ of the projection $\omega$ will be called the boundary of the prismatic shell. Such body will be called cusped (or tapered) body if the thickness of the body vanishes on some subset of the boundary $\partial \omega$ or on the whole one.

## 3. - Basic relations in the cylindrical system of coordinates <br> for the approximation $N=0$

From the basic relations in the cylindrical system of coordinates of the linear theory of elasticity, after integration with respect to $x_{3}$ within the limits $\stackrel{(-)}{b}\left(x_{1}, x_{2}\right)$ and $\stackrel{(+)}{b}\left(x_{1}, x_{2}\right)$, it is easy to derive the following basic relations in the zero approximation (see [5], pp. 27, 28, 149):

1. The equilibrium equations

$$
\begin{align*}
& \frac{1}{r} \frac{\partial \tau_{r \psi}^{0}}{\partial \psi}+\frac{\partial 0^{\sigma_{r}}}{\partial r}+\frac{0_{r}-0_{\sigma}^{\sigma}}{r}=0 \\
& \frac{1}{r} \frac{\partial \sigma_{\psi \psi}^{0}}{\partial \psi}+\frac{\partial \tau_{r \psi}}{\partial r}+\frac{2 \tau_{r \psi}^{0}}{r}=0  \tag{3.1}\\
& \frac{1}{r} \frac{\partial Z_{\psi \psi}}{\partial \psi}+\frac{\partial Z_{r}}{\partial r}+\frac{Z_{r}}{r}=0
\end{align*}
$$

(here it is assumed that the upper and lower surfaces of the prismatic shell are unloaded and the volume forces are neglected);
2. the kinematic formulas

$$
\stackrel{0}{e}_{r}=2 h \frac{\partial(2 h)^{-1} \stackrel{0}{u}_{r}}{\partial r}, \quad \stackrel{0}{e}_{\psi}=\frac{2 h}{r} \frac{\partial(2 h)^{-1}{\stackrel{0}{u_{\psi}}}_{\partial \psi}^{\partial \psi}+\frac{0_{r}}{r}, \quad e_{33}^{0}=0, ~ ;, ~}{0}=
$$

$$
\begin{aligned}
& \stackrel{0}{e}_{e_{\psi 3}}=\frac{b}{r} \frac{\partial(2 h)^{-1} \stackrel{0}{u}_{3}}{\partial \psi}, \quad{\stackrel{0}{e}{ }_{3 r}=b}^{\partial \psi} \frac{\partial(2 h)^{-1} \stackrel{0}{u}_{3}}{\partial r}
\end{aligned}
$$

i.e.,

$$
\begin{align*}
\stackrel{0}{e}_{r}=2 h \frac{\partial v_{r}}{\partial r} & \quad 0_{\psi}=\frac{2 h}{r}\left(\frac{\partial v_{\psi}}{\partial \psi}+v_{r}\right), \quad e_{33}^{0}=0  \tag{3.2}\\
e_{r \psi}^{0} & =\frac{b}{r}\left(\frac{\partial v_{r}}{\partial \psi}+r \frac{\partial v_{\psi}}{\partial r}-v_{\psi}\right)  \tag{3.3}\\
e_{e_{\psi 3}}^{0} & =\frac{b}{r} \frac{\partial v_{3}}{\partial \psi}, \quad e_{3 r}^{0}=b \frac{\partial v_{3}}{\partial r} \tag{3.4}
\end{align*}
$$

where

$$
v_{r}(r, \psi):=\frac{0_{r}(r, \psi)}{2 b}, \quad v_{\psi}(r, \psi):=\frac{0_{u_{\psi}}(r, \psi)}{2 b}, \quad v_{3}(r, \psi):=\frac{\stackrel{0}{u}(r, \psi)_{0}^{2 b}}{2 b}
$$

3. Constitutive Relations (Hooke's law)

$$
\begin{equation*}
\stackrel{0}{e}_{r}=\frac{1-\sigma^{2}}{E} \stackrel{0}{\sigma}_{r}-\frac{(1+\sigma) \sigma}{E} \stackrel{0}{\sigma}_{\psi}, \quad \stackrel{0}{e}_{\psi}=\frac{1-\sigma^{2}}{E} \stackrel{0}{\sigma}_{\psi}-\frac{(1+\sigma) \sigma}{E} \stackrel{0}{\sigma}_{r}, \tag{3.5}
\end{equation*}
$$

$$
\begin{align*}
& { }_{e_{r y}}^{0}=\frac{\stackrel{0}{\tau}_{r \psi}}{2 \mu}, \quad \stackrel{0}{e}_{e_{\psi 3}}=\frac{\stackrel{Q}{Z}_{\psi}}{2 \mu}, \quad{ }_{e_{3 r}}^{0}=\frac{{ }_{Z}^{Z}}{2 \mu},  \tag{3.6}\\
& \stackrel{0}{Z}_{z}=\sigma\left(\stackrel{0}{\sigma}_{\psi}+\stackrel{0}{\sigma}_{r}\right),
\end{align*}
$$

where $\sigma$ is Poission's ratio, $E$ is Young's modulus, $\mu$ is the Lamé constant, $,{ }_{u},{ }_{u}, u_{\mu}, \ddot{u}_{3}$ are the zero moments of the displacement vector components, $\stackrel{0}{e}_{r},{ }^{0} e_{4}, \stackrel{0}{e_{r y}}, \stackrel{0}{e_{3 r}}, \stackrel{0}{e_{\psi 3}}, e_{33}^{0}$ are the zero moments of the deformation tensor components, and ${ }_{\sigma}^{\sigma_{r}}, 0_{\sigma_{\psi}}, 0_{\tau \mu}, \stackrel{0}{Z}_{r},{\underset{Z}{Z}}_{\psi}, \stackrel{0}{Z}_{3}$ are the zero moments of the stress tensor components in the cylindrical coordinates. E.g.,

$$
{\underset{u}{u}}_{0}(r, \psi):=\int_{\substack{(-) \\ b \\(r, \psi)}}^{\stackrel{(+)}{b}(r, \psi)} u_{r}\left(r, \psi, x_{3}\right) d x_{3}, \stackrel{0}{\sigma}_{r}(r, \psi):=\int_{\substack{(-) \\ b \\(r, \psi)}}^{\stackrel{(+)}{b}(r, \psi)} \sigma_{r}\left(r, \psi, x_{3}\right) d x_{3} .
$$

Let us note that in the zero approximation it is assumed that

$$
\begin{align*}
& u_{r}\left(r, \psi, x_{3}\right) \approx v_{r}(r, \psi):=\frac{0_{u_{r}}(r, \psi)}{2 b}, \quad u_{\psi}\left(r, \psi, x_{3}\right) \approx v_{\psi}(r, \psi):=\frac{0_{\psi}(r, \psi)}{2 h}  \tag{3.7}\\
& u_{3}\left(r, \psi, x_{3}\right) \approx v_{3}(r, \psi):=\frac{u_{3}(r, \psi)}{2 h} .
\end{align*}
$$

Similar assumptions are made for the stress and deformation tensor components.

From (3.2), after integration, taking into account (3.5), we have

$$
\begin{equation*}
v_{r}=\frac{1-\sigma^{2}}{E} \int_{r_{0}}^{r} \frac{\sigma_{r}}{2 h} d r-\frac{(1+\sigma) \sigma}{E} \int_{r_{0}}^{r} \frac{\stackrel{\sigma}{\psi}^{2}}{2 h} d r+g_{1}(\psi) \tag{3.8}
\end{equation*}
$$

$$
\begin{equation*}
v_{\psi}=\frac{1-\sigma^{2}}{E} r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 b} d \psi-\frac{(1+\sigma) \sigma}{E} r \int_{\psi_{0}}^{\psi^{\psi}} \frac{\sigma_{r}}{2 b} d \psi-\int_{\psi_{0}}^{\psi} v_{r} d \psi+g_{2}(r), \tag{3.9}
\end{equation*}
$$

where $r_{0} e^{i \psi_{0}} \in \omega$ is a fixed point; $g_{1}(\psi)$ and $g_{2}(r)$ are arbitrary functions.
Substituting (3.8) in (3.9), we obtain

$$
\begin{align*}
v_{\psi}= & \frac{1-\sigma^{2}}{E} r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 b} d \psi-\frac{(1+\sigma) \sigma}{E} r \int_{\psi_{0}}^{\psi} \frac{\sigma_{r}}{2 b} d \psi-\frac{1-\sigma^{2}}{E} \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\psi} \frac{\sigma_{r}^{0}}{2 b} d \psi  \tag{3.10}\\
& +\frac{(1+\sigma) \sigma}{E} \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 h} d \psi-\int_{\psi_{0}}^{\psi /} g_{1} d \psi+g_{2}(r) .
\end{align*}
$$

Now, substituting (3.8), (3.10) in (3.3) and taking into account the first of (3.6), we get

$$
\begin{aligned}
& -\frac{\stackrel{0}{\tau}_{r \mu}}{2 h \mu}+\frac{1}{r}\left[\frac{1-\sigma^{2}}{E} \int_{r_{0}}^{r} \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial \psi} d r-\frac{(1+\sigma) \sigma}{E} \int_{r_{0}}^{r} \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{\psi}}{\partial \psi} d r\right] \\
& +\frac{1-\sigma^{2}}{E} \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 b} d \psi+\frac{1-\sigma^{2}}{E} r \int_{\psi_{0}}^{\psi} \frac{\partial(2 h)^{-1}{ }^{0} \sigma_{\psi}}{\partial r} d \psi-\frac{(1+\sigma) \sigma}{E} \int_{\psi_{0}}^{\psi^{\prime}} \int_{\sigma_{r}}^{\sigma_{r}} d \psi \\
& -\frac{(1+\sigma) \sigma}{E} r \int_{\psi_{0}}^{\psi} \frac{\partial(2 h)^{-1}{ }^{0} \sigma_{r}}{\partial r} d \psi-\frac{1-\sigma^{2}}{E} \int_{\psi_{0}}^{\psi} \frac{\stackrel{\sigma}{r}_{r}}{2 b} d \psi+\frac{(1+\sigma) \sigma}{E} \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 b} d \psi \\
& -\frac{1-\sigma^{2}}{E} \int_{\psi_{0}}^{\psi /} \frac{\sigma_{\psi}}{2 h} d \psi+\frac{(1+\sigma) \sigma}{E} \int_{\psi_{0}}^{\psi /} \frac{\sigma_{r}}{2 b} d \psi+\frac{1-\sigma^{2} 1}{E} \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\psi /} \frac{\sigma_{r}}{2 b} d \psi \\
& -\frac{(1+\sigma) \sigma}{E} \frac{1}{r} \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 b} d \psi=-g_{2}^{\prime}(r)-\frac{1}{r} \int_{\psi_{0}}^{\psi} g_{1} d \psi+\frac{1}{r} g_{2}(r)-\frac{1}{r} g_{1}^{\prime}(\psi) .
\end{aligned}
$$

Combining in (3.11) like terms and multiplying both the sides of (3.11) by $\frac{E}{1+\sigma} r$,
because of $\mu=\frac{E}{2(1+\sigma)}$, we have

$$
\begin{align*}
& -2 \frac{r_{\tau_{r \psi}}^{0}}{2 h}+(1-\sigma) \int_{r_{0}}^{r} \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial \psi} d r-\sigma \int_{r_{0}}^{r} \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{\psi}}{\partial \psi} d r \\
& \quad+(1-\sigma) r^{2} \int_{\psi_{0}}^{\psi /} \frac{\partial(2 h)^{-1} \stackrel{\sigma}{\sigma}_{\psi}^{0}}{\partial r} d \psi-\sigma r^{2} \int_{\psi_{0}}^{\psi} \frac{\partial(2 h)^{-1} \stackrel{\sigma}{\sigma}_{r}^{0}}{\partial r} d \psi-(1-\sigma) r \int_{\psi_{0}}^{\psi} \frac{\sigma_{r}}{2 b} d \psi \\
& \quad+\sigma r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 h} d \psi+(1-\sigma) \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\sigma_{\psi}} \frac{\sigma_{r}}{2 h} d \psi-\sigma \int_{r_{0}}^{r} d r \int_{\psi_{0}}^{\psi} \frac{\sigma_{\psi}}{2 h} d \psi  \tag{3.12}\\
& \quad=\frac{E}{1+\sigma}\left[-\int_{\psi_{0}}^{\psi} g_{1} d \psi-g_{1}^{\prime}(\psi)-r g_{2}^{\prime}(r)+g_{2}(r)\right]
\end{align*}
$$

But (3.12) holds if and only if the left hand side is representable as a sum of two functions, when one of them depends only on $r$ and the another one depends only on $\psi$. In other words the second order mixed derivative of the left hand side should equal to zero:

$$
\begin{align*}
& -2 \frac{\partial^{2} r(2 h)^{-1} \stackrel{0}{\tau}_{r \psi}}{\partial r \partial \psi}+(1-\sigma) \frac{\partial^{2}(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial \psi^{2}}-\sigma \frac{\partial^{2}(2 h)^{-1} \stackrel{0}{\sigma}_{\psi}}{\partial \psi^{2}} \\
& +(2-\sigma) r \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{\psi}}{\partial r}+(1-\sigma) r^{2} \frac{\partial^{2}(2 h)^{-1} \stackrel{0}{\sigma}_{\psi}}{\partial r^{2}}  \tag{3.13}\\
& -\sigma r^{2} \frac{\partial^{2}(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial r^{2}}-(1+\sigma) r \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial r}=0 .
\end{align*}
$$

From (3.4), (3.6) there follows

$$
\frac{\partial v_{3}}{\partial \psi}=\frac{r \stackrel{0}{Z}_{\psi}}{2 h \mu}, \quad \frac{\partial v_{3}}{\partial r}=\frac{\stackrel{0}{Z}_{r}}{2 h \mu}
$$

Hence, the necessary and sufficient condition for restoration of $v_{3}$ by its derivatives is

$$
\begin{equation*}
\frac{\partial}{\partial r}\left(\frac{r 0_{\psi}}{2 h \mu}\right)=\frac{\partial}{\partial \psi}\left(\frac{0_{Z_{r}}}{2 h \mu}\right) \tag{3.14}
\end{equation*}
$$

Equations (3.13), (3.14) are compatibility equations, i.e., analogous of BeltramiMichell compatibility equations in the case under consideration.

## 4. - The title problem

Let the projection $\omega$ of the prismatic shell-like body with the thickness

$$
\begin{equation*}
2 h\left(x_{1}, x_{2}\right)=h_{0} x_{2}^{\kappa}=h_{0} r^{\kappa} \sin ^{\kappa} \psi, \quad h_{0}=\text { const }>0, \quad \kappa=\text { const } \geq 0 \tag{4.1}
\end{equation*}
$$



Fig. 1.
be the less angle between rays $\psi=\beta$ and $\psi=\pi-\beta, 0 \leq \beta<\frac{\pi}{2}$ (see Fig. 1). Let further the body be loaded at the vertex of the angle by the concentrated force $\left(-S_{1},-S_{2},-S_{3}\right)_{R}$. $R$ means the vector components in the system $O x_{1} x_{2} x_{3}$. We are looking for the solution of the problem in the zero approximation in the following form

$$
\begin{align*}
& \stackrel{0}{\sigma}_{r}=\varphi_{\kappa}(\psi) \frac{\sin ^{\kappa} \psi}{r}, \quad \stackrel{0}{Z}_{r}=k \frac{\sin ^{\kappa} \psi}{r}, \quad \stackrel{0}{\sigma}_{\psi}=\stackrel{0}{\tau}_{r \psi}=\stackrel{0}{Z}_{\psi \psi}=0, \quad \stackrel{0}{\tau}_{r \psi} \equiv \stackrel{0}{\tau}_{\psi r},  \tag{4.2}\\
& \quad{ }_{Z}^{Z}=\sigma\left(\stackrel{0}{\sigma}_{\psi}+\stackrel{0}{\sigma}_{r}\right)=\sigma \stackrel{0}{\sigma}_{r}, \quad \kappa \geq 0
\end{align*}
$$

where

$$
\varphi_{\kappa}(\psi):= \begin{cases}\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi} & \text { when } \kappa>\frac{1}{v}  \tag{4.3}\\ \gamma_{2}+\delta_{2} \psi & \text { when } \kappa=\frac{1}{v} \\ \gamma_{3} \cos (c \psi)+\delta_{3} \sin (c \psi) & \text { when } \frac{1}{v}>\kappa \geq 0\end{cases}
$$

$$
\begin{equation*}
a:=\sqrt{(\kappa+1)(v \kappa-1)}, \quad c:=\sqrt{(\kappa+1)(1-v \kappa)}, \quad v:=\frac{\sigma}{1-\sigma} \quad(0<\sigma<1) \tag{4.4}
\end{equation*}
$$

and constants $k, \gamma_{i}, \delta_{i}, i=1,2,3$, should be determined. It is easy to see that functions (4.2) satisfy (3.1), (3.14), and (3.13), in view of (4.4).

Indeed, since

$$
\stackrel{0}{\sigma}_{\psi \psi}=0, \quad \stackrel{0}{\tau}_{r \mu}=0, \quad \stackrel{0}{Z}_{\psi}=0
$$

equations (3.1), (3.14), and (3.13) we can rewrite as follows

$$
\frac{\partial{\stackrel{0}{\sigma_{r}}}_{r}}{\partial r}+\frac{0_{r}}{r}=0, \quad \frac{\partial \stackrel{0}{Z}_{r}}{\partial r}+\frac{\stackrel{0}{Z}_{r}}{r}=0, \quad \frac{\partial}{\partial \psi}\left(\frac{0_{Z}^{Z_{r}}}{2 h}\right)=0
$$

and

$$
(1-\sigma) \frac{\partial^{2}(2 b)^{-1} \stackrel{0}{\sigma}_{r}}{\partial \psi^{2}}-\sigma r^{2} \frac{\partial^{2}(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial r^{2}}-(1+\sigma) r \frac{\partial(2 h)^{-1} \stackrel{0}{\sigma}_{r}}{\partial r}=0
$$

respectively. Evidently, the first group of equations is satisfied by

$$
\stackrel{0}{\sigma}_{r}=\varphi_{\kappa}(\psi) \frac{\sin ^{\kappa} \psi}{r}, \quad \stackrel{0}{Z}_{r}=k \frac{\sin ^{\kappa} \psi}{r}
$$

The last equation holds as well for

$$
\frac{\stackrel{0}{\sigma}_{r}}{2 b}=\varphi_{\kappa}(\psi) \frac{1}{h_{0} r^{\kappa+1}}
$$

because of (see (4.4))

$$
\begin{aligned}
& (1-\sigma) a^{2}-\sigma(\kappa+1)(\kappa+2)+(1+\sigma)(\kappa+1)=(1-\sigma)(\kappa+1)\left(\kappa \frac{\sigma}{1-\sigma}-1\right) \\
& \quad+(\kappa+1)(1-\sigma-\sigma \kappa)=0 \\
& -\sigma(\kappa+1)(\kappa+2)+(1+\sigma)(\kappa+1)=(\kappa+1)(1-\sigma \kappa-\sigma) \\
& \quad=\left(\frac{1-\sigma}{\sigma}+1\right)\left(1-\sigma \frac{1-\sigma}{\sigma}-\sigma\right)=0 \\
& -(1-\sigma) c^{2}-\sigma(\kappa+1)(\kappa+2)+(1+\sigma)(\kappa+1)=-(1-\sigma)(\kappa+1)\left(1-\kappa \frac{\sigma}{1-\sigma}\right) \\
& \quad+(\kappa+1)(1-\sigma-\sigma \kappa)=0
\end{aligned}
$$

$$
\text { for } \kappa>\frac{1}{v}, \kappa=\frac{1}{v}=\frac{1-\sigma}{\sigma}, \frac{1}{v}>\kappa \geq 0 \text {, correspondingly. }
$$

The homogeneous boundary conditions on $\psi=\beta$ and $\psi=\pi-\beta(r>0)$ :

$$
\stackrel{0}{\sigma}_{\psi \psi}=0, \quad \stackrel{0}{\tau}_{\tau \psi}=0, \quad \stackrel{0}{Z}_{\psi \psi}=0
$$

are obviously fulfilled by (4.2). The constants $k, \gamma_{i}, \delta_{i}, i=1,2,3$, we have to calculate from the following condition: the stresses distributed on any cylindrical surface of the radius $r$ lying in the body should be equivalent to the concentrated force $\left(S_{1}, S_{2}, S_{3}\right)_{R}$. On the surface element corresponding to the angle $d \psi$ acts the force

$$
\left(\stackrel{0}{\sigma}_{r}, \stackrel{0}{\tau}_{\psi r} \equiv 0, \stackrel{0}{Z_{r}}\right)_{C} r d \psi
$$

where $C$ means the components in the cylindrical system of coordinates. Let us note that

$$
(1,0,0)_{R}=(\cos \psi,-\sin \psi, 0)_{C}, \quad(0,1,0)_{R}=(\sin \psi, \cos \psi, 0)_{C}, \quad(0,0,1)_{R}=(0,0,1)_{C}
$$

Projecting the forces distributed on any cylindrical surface of the radius $r$, lying in the angle between the rays $\psi=\beta$ and $\psi=\pi-\beta$, on the axes $x_{1}, x_{2}, x_{3}$, and then integrating with respect to $\psi$ from $\beta$ to $\pi-\beta$, we get the components of the resultant force of the above forces:

$$
\begin{equation*}
\int_{\beta}^{\pi-\beta} \stackrel{0}{\sigma}_{r} r \cos \psi d \psi=S_{1}, \quad \int_{\beta}^{\pi-\beta} \stackrel{0}{\sigma}_{r} r \sin \psi d \psi=S_{2}, \quad \int_{\beta}^{\pi-\beta} Z_{r} r d \psi=S_{3} . \tag{4.5}
\end{equation*}
$$

Therefore, substituting (4.2) in (4.5):

1. If $\kappa>\frac{1}{v}$ :

$$
\begin{gather*}
\gamma_{1} \int_{\beta}^{\pi-\beta} e^{a \psi \psi} \sin ^{\kappa} \psi \cos \psi d \psi+\delta_{1} \int_{\beta}^{\pi-\beta} e^{-a \psi} \sin ^{\kappa} \psi \cos \psi d \psi=S_{1}  \tag{4.6}\\
\gamma_{1} \int_{\beta}^{\pi-\beta} e^{a \psi} \sin ^{\kappa+1} \psi d \psi+\delta_{1} \int_{\beta}^{\pi-\beta} e^{-a \psi} \sin ^{\kappa+1} \psi d \psi=S_{2} . \tag{4.7}
\end{gather*}
$$

Let

$$
\stackrel{\beta}{\Lambda}(a, b):=\int_{\beta}^{\pi-\beta} e^{a \psi} \sin ^{-b} \psi d \psi, \quad \Lambda^{*}(a, b)=\int_{\beta}^{\pi-\beta} e^{a \psi} \sin ^{-b} \psi \cos \psi d \psi .
$$

The determinant $\Delta_{1}$ of the system (4.6), (4.7) has the form

$$
\begin{aligned}
\Delta_{1} & =\Lambda^{\beta}(a,-\kappa){ }^{\beta}(-a,-\kappa-1)-{ }_{\Lambda}^{\beta}(a,-\kappa-1)^{\beta} \Lambda^{*}(-a,-\kappa) \\
& =\Lambda^{\beta}(a,-\kappa) e^{-a \pi} \Lambda^{\beta}(a,-\kappa-1)+{ }_{\Lambda}^{\beta}(a,-\kappa-1) e^{-a \pi} \Lambda^{*}(a,-\kappa) \\
& =2 e^{-a \pi} \Lambda_{\Lambda}^{\beta}(a,-\kappa-1)^{\beta} \Lambda^{*}(a,-\kappa)
\end{aligned}
$$

since

$$
\begin{align*}
e^{a \pi}{ }_{\Lambda}^{\beta}(-a, b) & =-\int_{\beta}^{\pi-\beta} e^{a(\pi-\psi)} \sin ^{-b}(\pi-\psi) d(\pi-\psi)=-\int_{\pi-\beta}^{\beta} e^{a \tau} \sin ^{-b} \tau d \tau=\Lambda(a, b), \\
e^{a \pi} \Lambda^{\beta}(-a, b) & =\int_{\beta}^{\pi-\beta} e^{a(\pi-\psi)} \sin ^{-b}(\pi-\psi) \cos (\pi-\psi) d(\pi-\psi)  \tag{4.8}\\
& =\int_{\pi-\beta}^{\beta} e^{a \tau} \sin ^{-b} \tau \cos \tau d \tau=-\stackrel{\beta}{\Lambda}(a, b) .
\end{align*}
$$

Note that ${ }^{\beta} \Lambda^{*}(0,-\kappa)=0$. But in the case under consideration $a>0$. Therefore, by virtue of the mean value theorem of the integral calculus for the fixed $\left.\psi_{0} \in\right] \beta ; \pi-\beta[$, evidently,

$$
\begin{gathered}
\Lambda^{\beta}(a,-\kappa)=\int_{\beta}^{\pi-\beta} e^{a \psi \psi} \sin ^{\kappa} \psi d \sin \psi=\frac{e^{a(\pi-\beta)}-e^{a \beta}}{\kappa+1} \sin ^{\kappa+1} \beta-\frac{a}{\kappa+1} \int_{\beta}^{\pi-\beta} e^{a \psi \psi} \sin ^{\kappa+1} \psi d \psi \\
=\frac{e^{a(\pi-\beta)}-e^{a \beta}}{\kappa+1} \sin ^{\kappa+1} \beta-\frac{a}{\kappa+1} \sin ^{\kappa+1} \psi \psi_{0} \int_{\beta}^{\pi-\beta} e^{a \psi \psi} d \psi \\
=\frac{e^{a(\pi-\beta)}-e^{a \beta}}{\kappa+1}\left(\sin ^{\kappa+1} \beta-\sin ^{\kappa+1} \psi_{0}\right) \neq 0
\end{gathered}
$$

because of $\psi_{0} \neq \beta, \pi-\beta$. Thus, $\Delta_{1} \neq 0$.

Taking into account (4.8) and solving the system (4.6), (4.7) with respect to $\gamma_{1}, \delta_{1}$, we have

$$
\begin{equation*}
\gamma_{1}=\frac{S_{1}{ }^{\beta}(a,-\kappa-1)-S_{2}{ }^{\beta} \Lambda^{*}(a,-\kappa)}{e^{a \pi} \Delta_{1}} \tag{4.9}
\end{equation*}
$$

$$
\begin{equation*}
\delta_{1}=\frac{S_{2}^{\beta} \Lambda^{*}(a,-\kappa)-S_{1}{ }^{\beta}{ }^{\Lambda}(a,-\kappa-1)}{\Delta_{1}} . \tag{4.10}
\end{equation*}
$$

2. If $a=\frac{1}{v}$ :

$$
\begin{equation*}
\gamma_{2} \int_{\beta}^{\pi-\beta} \sin ^{\kappa} \psi \cos \psi d \psi+\delta_{2} \int_{\beta}^{\pi-\beta} \psi \sin ^{\kappa} \psi \cos \psi d \psi=S_{1} \tag{4.11}
\end{equation*}
$$

$$
\begin{equation*}
\gamma_{2} \int_{\beta}^{\pi-\beta} \sin ^{\kappa+1} \psi d \psi+\delta_{2} \int_{\beta}^{\pi-\beta} \psi \sin ^{\kappa+1} \psi d \psi=S_{2} \tag{4.12}
\end{equation*}
$$

Let

$$
\begin{align*}
& \stackrel{\beta}{\Lambda}(b):=\int_{\beta}^{\pi-\beta} \psi \sin ^{-b} \psi \cos \psi d \psi,  \tag{4.13}\\
& \stackrel{\beta}{\wedge}(b):=\int_{\beta}^{\pi-\beta} \psi \sin ^{-b} \psi d \psi .
\end{align*}
$$

Evidently, the first summand of (4.11) is equal to zero, therefore,

$$
\begin{equation*}
\delta_{2}=\frac{S_{1}}{\Lambda(-\kappa)} . \tag{4.14}
\end{equation*}
$$

Substituting the latter in (4.12) and solving the obtained equation with respect to $\gamma_{2}$, we get

$$
\begin{equation*}
\gamma_{2}=\frac{S_{2} \stackrel{\beta}{\Lambda}(-\kappa)-S_{1} \stackrel{\beta}{\wedge}(-\kappa-1)}{\beta} \stackrel{\beta}{\Lambda}(0,-\kappa-1)_{\wedge}^{\Lambda}(-\kappa) . \tag{4.15}
\end{equation*}
$$

Obviously, denominators of (4.14), (4.15) are not zero. Indeed, by virtue of the mean value theorem of integral calculus for the fixed $\left.\psi_{0} \in\right] \beta ; \pi-\beta[$, evidently,

$$
\begin{aligned}
{ }_{A}^{\beta}(-\kappa) & =\int_{\beta}^{\pi-\beta} \psi \sin ^{\kappa} \psi d \sin \psi=\frac{\pi-2 \beta}{\kappa+1} \sin ^{\kappa+1} \beta-\frac{1}{\kappa+1} \int_{\beta}^{\pi-\beta} \sin ^{\kappa+1} \psi d \psi \\
& =\frac{\pi-2 \beta}{\kappa+1} \sin ^{\kappa+1} \beta-\frac{\pi-2 \beta}{\kappa+1} \sin ^{\kappa+1} \psi_{0} \neq 0 .
\end{aligned}
$$

3. If $\frac{1}{v}>\kappa \geq 0$ :

$$
\begin{equation*}
\gamma_{3} \int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{\kappa} \psi \cos \psi d \psi+\delta_{3} \int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{\kappa} \psi \cos \psi d \psi=S_{1}, \tag{4.16}
\end{equation*}
$$

$$
\gamma_{3} \int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{\kappa+1} \psi d \psi+\delta_{3} \int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{\kappa+1} \psi d \psi=S_{2}
$$

Let

$$
\begin{array}{ll}
\beta(c, b):=\int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{-b} \psi d \psi, & A^{*}(c, b):=\int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{-b} \psi \cos \psi d \psi, \\
\frac{\beta}{B}(c, b):=\int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{-b} \psi d \psi, & B^{*}(c, b):=\int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{-b} \psi \cos \psi d \psi .
\end{array}
$$

Evidently,

$$
\begin{align*}
& \cos \frac{c \pi}{2} A^{\beta}(c, b)+\sin \frac{c \pi}{2}{ }^{\beta} *(c, b)=\cos \frac{c \pi}{2} \int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{-b} \psi \cos \psi d \psi \\
& +\sin \frac{c \pi}{2} \int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{-b} \psi \cos \psi d \psi=\int_{\beta}^{\pi-\beta} \cos \left[c\left(\psi-\frac{\pi}{2}\right)\right] \sin ^{-b} \psi \cos \psi d \psi \\
& =\int_{\beta-\frac{\pi}{2}}^{\frac{\pi}{2}-\beta} \cos (c \tau) \sin ^{-b}\left(\frac{\pi}{2}+\tau\right) \cos \left(\frac{\pi}{2}+\tau\right) d \tau=-\int_{\beta-\frac{\pi}{2}}^{\frac{\pi}{2}-\beta} \cos (c \tau) \cos ^{-b} \tau \sin \tau d \tau=0  \tag{4.18}\\
& \cos \frac{c \pi}{2} \stackrel{\beta}{B}(c, b)-\sin \frac{c \pi}{2} A_{A}^{\beta}(c, b)=\cos \frac{c \pi}{2} \int_{\beta}^{\pi-\beta} \sin (c \psi) \sin ^{-b} \psi d \psi \\
& -\sin \frac{c \pi}{2} \int_{\beta}^{\pi-\beta} \cos (c \psi) \sin ^{-b} \psi d \psi=\int_{\beta}^{\pi-\beta} \sin \left[c\left(\psi-\frac{\pi}{2}\right)\right] \sin ^{-b} \psi d \psi \\
& =\int_{\beta-\frac{\pi}{2}}^{\frac{\pi}{2}-\beta} \sin (c \psi) \cos { }^{-b} \psi d \psi=0,
\end{align*}
$$

since the last integrals in both the expressions are the integrals along a symmetric interval
with odd integrands. Therefore, we have $\stackrel{\beta}{A}(2 k+1, b)=0, \quad \stackrel{\beta}{B}(2 k, b)=0, \quad \stackrel{\beta}{B^{*}}(2 k+1, b)=0, \quad{ }^{\beta}{ }^{*}(2 k, b)=0$ for $k=1,2, \ldots$, under the additional restriction $b<1$ in case $\beta=0$.

The determinant of the system (4.16), (4.17) has the form

$$
\Delta_{3}=\stackrel{\beta}{A^{*}}(c,-\kappa) \stackrel{\beta}{B}(c,-\kappa-1)-\stackrel{\beta}{A}(c,-\kappa-1)^{\beta} *(c,-\kappa) .
$$

If $c=2 k \quad(k=1,2, \ldots)$, then

$$
\Delta_{3}=-\stackrel{\beta}{B}^{*}(2 k,-\kappa) \stackrel{\beta}{A}(2 k,-\kappa-1) \neq 0 .
$$

If $c \neq 2 k(k=1,2, \ldots)$, then

$$
\begin{aligned}
\Delta_{3}= & \stackrel{\beta}{B}(c,-\kappa-1){ }^{\beta} A^{*}(c,-\kappa)+\operatorname{ctg} \frac{c \pi}{2}{ }^{\beta} A^{*}(c,-\kappa) \operatorname{ctg} \frac{c \pi}{2} \stackrel{\beta}{B}(c,-\kappa-1) \\
& =\left(1+\operatorname{ctg}^{2} \frac{c \pi}{2}\right){ }^{\beta} A^{*}(c,-\kappa) \stackrel{\beta}{B}(c,-\kappa-1)=\sin ^{-2} \frac{c \pi}{2}{ }^{\beta} A^{*}(c,-\kappa) \stackrel{\beta}{B}(c,-\kappa-1) \neq 0,
\end{aligned}
$$

because of

$$
\stackrel{\beta}{A}(c, b)=\operatorname{ctg} \frac{c \pi}{2}{ }_{B}^{\beta}(c, b), \quad{ }_{B}^{\beta}(c, b)=-\operatorname{ctg} \frac{c \pi}{2}{ }^{\beta} A^{*}(c, b)
$$

which there follow from (4.18)
Solving the system (4.16), (4.17), we obtain

$$
\begin{align*}
\gamma_{3} & =\frac{S_{1}{ }^{\beta}(c,-\kappa-1)-S_{2}{ }^{\beta}{ }^{*}(c,-\kappa)}{\Delta_{3}},  \tag{4.19}\\
\delta_{3} & =\frac{S_{2}{ }^{\beta}{ }^{*}(c,-\kappa)-S_{1}{ }^{\beta}(c,-\kappa-1)}{\Delta_{3}} . \tag{4.20}
\end{align*}
$$

In all the above cases, evidently,

$$
\begin{equation*}
k=\frac{S_{3}}{\beta(0,-\kappa)} \tag{4.21}
\end{equation*}
$$

Remark 4.1: On the one hand,

$$
2 h\left(x_{1}, x_{2}\right)=O\left(r^{\kappa}\right) \text {, as } r \rightarrow \infty \text { for } \kappa \geq 0 .
$$

Hence, for $\kappa>0$ the thickness of the prismatic shell-like body tends to infinity as $r \rightarrow \infty$. On the other hand, as we see from (4.2),

$$
\stackrel{0}{\sigma}_{r}=O\left(\frac{1}{r}\right), \stackrel{0}{Z}_{r}=O\left(\frac{1}{r}\right), \text { as } r \rightarrow \infty
$$

Therefore, for sufficiently large $r$ the zero moments of stresses are arbitrarily small, and
actually, we can assume that in the non-thin part of the body under consideration we have not stressed (strained) state. Evidently, to this end $h_{0}$ should be chosen duely.

Remark 4.2: Let $\kappa=0$ (i.e., the prismatic shell-like body under consideration becomes a plate of the constant thickness), $S_{1}=0, S_{2} \neq 0, S_{3}=0$, and the angle be equal to $\pi$ (i.e., $\beta=0$ ). Then the solution (4.2) with (4.3), (4.19), (4.20) coincides with the wellknown solution of the classical Flamant problem [1]. The same expressions give the wellknown solution for $0<\beta<\frac{\pi}{2}$ when either $S_{1}=0, S_{2} \neq 0, S_{3}=0$, or $S_{1} \neq 0, S_{2}=0$, $S_{3}=0$ (see, e.g., [6], pp. 107, 108, and references therein or [7], pp. 516-518).

Let us, now, establish explicit expressions for the displacement vector components $v_{r}$, $v_{\psi}, v_{3}$. To this end we need again to consider the cases

$$
\kappa>\frac{1}{v}, \kappa=\frac{1}{v}, \text { and } \frac{1}{v}>\kappa>0
$$

separately. Also the case $\kappa=0$ should be considered separately but it is classical and wellknown one (see, e.g., [7], p. 518).

1. $\kappa>\frac{1}{v}$.

According to (3.8), (3.10), (4.2), (4.3), (4.1) after integration, we obtain

$$
\begin{align*}
v_{r} & =\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right)\left(r^{-\kappa}-r_{0}^{-\kappa}\right)+g_{1}(\psi)  \tag{4.22}\\
& =\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right) r^{-\kappa}+\Phi_{1}(\psi),
\end{align*}
$$

where

$$
\begin{align*}
& \Phi_{1}(\psi):=-\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right) r_{0}^{-\kappa}+g_{1}(\psi) \\
& v_{\psi}=-\frac{(1+\sigma) \sigma}{a h_{0} E}\left[\gamma_{1} e^{a \psi}-\delta_{1} e^{-a \psi}-\left(\gamma_{1} e^{a \psi_{0}}-\delta_{1} e^{-a \psi_{0}}\right)\right] r^{-\kappa} \\
&+\frac{\left(1-\sigma^{2}\right)}{a \kappa h_{0} E}\left[\gamma_{1} e^{a \psi}-\delta_{1} e^{-a \psi}-\left(\gamma_{1} e^{a \psi \psi_{0}}-\delta_{1} e^{-a \psi_{0}}\right)\right] r^{-\kappa} \\
&-\int_{\psi_{0}}^{\psi}\left[\frac{\left(1-\sigma^{2}\right)}{\kappa h_{0} E} r_{0}^{-\kappa}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi \psi}\right)+g_{1}(\psi)\right] d \psi+g_{2}(r)  \tag{4.23}\\
&= \frac{(1+\sigma)(1-\sigma-\kappa \sigma)}{\kappa a E h_{0}}\left(\gamma_{1} e^{a \psi \psi}-\delta_{1} e^{-a \psi}\right) r^{-\kappa}-\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi+\Phi_{2}(r),
\end{align*}
$$

where

$$
\Phi_{2}(r):=-\frac{(1+\sigma)(1-\sigma-\kappa \sigma)}{\kappa a E h_{0}}\left(\gamma_{1} e^{a \psi_{0}}-\delta_{1} e^{-a \psi_{0}}\right) r^{-\kappa}+g_{2}(r) .
$$

Since $0_{r \psi}^{0}=0$, from (3.6) we conclude

$$
\begin{equation*}
\stackrel{0}{e}_{r \mu}=0 \tag{4.24}
\end{equation*}
$$

i. e., by virtue of (3.3),

$$
\begin{equation*}
\frac{\partial v_{r}}{\partial \psi}+r \frac{\partial v_{\psi}}{\partial r}-v_{\psi}=0 \tag{4.25}
\end{equation*}
$$

Substituting (4.22), (4.23) in (4.25) and assuming $\Phi_{a} \in C^{1}, a=1,2$, we get

$$
\begin{align*}
& -\frac{1+\sigma}{\kappa a E h_{0}} r^{-\kappa}\left(\gamma_{1} e^{a \psi}-\delta_{1} e^{-a \psi}\right)\left[(1-\sigma) a^{2}+\kappa(1-\kappa \sigma-\sigma)+1-\kappa \sigma-\sigma\right] \\
& \quad+\Phi_{1}^{\prime}(\psi)+\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi+r \Phi_{2}^{\prime}(r)-\Phi_{2}(r)=0 \tag{4.26}
\end{align*}
$$

But, in view of (4.4),

$$
\begin{align*}
& (1-\sigma) a^{2}+(1+\kappa)(1-\kappa \sigma-\sigma) \\
& \quad=(1-\sigma)(1+\kappa)\left(\kappa \frac{\sigma}{1-\sigma}-1\right)+(1+\kappa)(1-\kappa \sigma-\sigma)=0 \tag{4.27}
\end{align*}
$$

Hence, from (4.26), (4.27) we arrive at

$$
\Phi_{1}^{\prime}(\psi)+\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi=-r \Phi_{2}^{\prime}(r)+\Phi_{2}(r) .
$$

Whence,

$$
\begin{gather*}
\Phi_{1}^{\prime}(\psi)+\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi=-C_{1},  \tag{4.28}\\
r \Phi_{2}^{\prime}(r)-\Phi_{2}(r)=C_{1} \tag{4.29}
\end{gather*}
$$

where $C_{1}=$ const.
Let $\Phi_{1} \in C^{2}$, then from (4.28) we have

$$
\Phi_{1}^{\prime \prime}(\psi)+\Phi_{1}(\psi)=0
$$

The general solution of the last equation has the following form

$$
\begin{equation*}
\Phi_{1}(\psi)=C_{2} \cos \psi+C_{3} \sin \psi, \quad C_{2}, C_{3}=\text { const } . \tag{4.30}
\end{equation*}
$$

Evidently, (4.30) will be the general solution of the integro-differential equation (4.28) if the constants $C_{i}, i=1,2,3$ and $\psi_{0}$ are such that

$$
\begin{equation*}
C_{3} \cos \psi_{0}-C_{2} \sin \psi_{0}+C_{1}=0 \tag{4.31}
\end{equation*}
$$

The general solution of equation (4.29) has the following form

$$
\begin{equation*}
\Phi_{2}(r)=C_{4} r-C_{1}, \quad C_{4}=\text { const } . \tag{4.32}
\end{equation*}
$$

But the obtained expressions for $\Phi_{1}(\psi)$ and $\Phi_{2}(\psi)$ correspond to the rigid motion. Indeed, substituting

$$
\begin{gathered}
v_{r}=C_{2} \cos \psi+C_{3} \sin \psi, \\
v_{\psi}=-\int_{\psi_{0}}^{\psi}\left(C_{2} \cos \psi+C_{3} \sin \psi\right) d \psi+C_{4} r-C_{1},
\end{gathered}
$$

under the condition (4.31), in (3.2), (3.3), we obtain

$$
\stackrel{0}{e}_{r}=0, \quad \stackrel{0}{e}_{\psi}=0, \quad \stackrel{0}{e}_{r \psi}=\frac{h_{0} x_{2}^{k}}{2 r}\left(-C_{2} \sin \psi_{0}+C_{3} \cos \psi_{0}+C_{1}\right)=0
$$

Thus, assuming $C_{i}=0, i=1,2,3,4$, up to the rigid motion we have

$$
\begin{align*}
& v_{r}=\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right) r^{-\kappa}  \tag{4.33}\\
& v_{\psi}=\frac{(1+\sigma)(1-\kappa \sigma-\sigma)}{\kappa a E h_{0}}\left(\gamma_{1} e^{a \psi}-\delta_{1} e^{-a \psi}\right) r^{-\kappa} .
\end{align*}
$$

For the zero moments of the deformation tensor components, substituting (4.33) in (3.2), (3.3) and taking into account (4.1), we get

$$
\begin{aligned}
& \stackrel{0}{e}_{r}=\frac{1-\sigma^{2}}{E}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right) \frac{\sin ^{\kappa} \psi}{r}, \\
& { }_{e_{\psi}}^{0}=-\frac{(1+\sigma) \sigma}{E}\left(\gamma_{1} e^{a \psi}+\delta_{1} e^{-a \psi}\right) \frac{\sin ^{\kappa} \psi}{r}, \\
& e_{r \psi}=0 .
\end{aligned}
$$

2. $\kappa=\frac{1}{v}$.

According to (3.8), (3.10), (4.2), (4.3), similarly to the first case, we obtain

$$
\begin{align*}
& v_{r}=\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{2}+\delta_{2} \psi\right) r^{-\kappa}+\Phi_{1}(\psi) \\
& v_{\psi}=-\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi+\Phi_{2}(r) \tag{4.34}
\end{align*}
$$

where $\Phi_{1}(\psi)$ and $\Phi_{2}(r)$ are arbitrary functions of their arguments. Let $\Phi_{1} \in C^{2}$ and $\Phi_{2} \in C^{1}$. Substituting (4.34) in (4.25), we have

$$
\frac{\sigma^{2}-1}{\kappa E h_{0}} \delta_{2} r^{-\kappa}+\Phi_{1}^{\prime}(\psi)+r \Phi_{2}^{\prime}(r)+\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi-\Phi_{2}(r)=0 .
$$

Whence,

$$
\begin{gather*}
\Phi_{1}^{\prime}(\psi)+\int_{\psi_{0}}^{\psi} \Phi_{1}(\psi) d \psi=-C_{1},  \tag{4.35}\\
r \Phi_{2}^{\prime}(r)-\Phi_{2}(r)=C_{1}+\frac{1-\sigma^{2}}{\kappa E h_{0}} \delta_{2} r^{-\kappa} . \tag{4.36}
\end{gather*}
$$

Integro-differential equation (4.35) we have already solved (see (4.30), (4.31)). The general solution of (4.36) is

$$
\Phi_{2}(r)=C_{4} r-C_{1}-\frac{1-\sigma^{2}}{\kappa(1+\kappa) E h_{0}} \delta_{2} r^{-\kappa}
$$

Since under the condition (4.31) the expressions

$$
\begin{aligned}
& v_{r}=C_{2} \cos \psi+C_{3} \sin \psi \\
& v_{\psi}=\int_{\psi_{0}}^{\psi}\left(C_{2} \cos \psi+C_{3} \sin \psi\right) d \psi+C_{4} r-C_{1}
\end{aligned}
$$

correspond to the rigid motion. Thus, up to the rigid motion we get

$$
\begin{align*}
& v_{r}=\frac{\sigma^{2}-1}{\kappa E h_{0}}\left(\gamma_{2}+\delta_{2} \psi\right) r^{-\kappa},  \tag{4.37}\\
& v_{\psi}=\frac{\sigma^{2}-1}{\kappa(\kappa+1) E h_{0}} \delta_{2} r^{-\kappa}
\end{align*}
$$

Substituting (4.37) in (3.2), (3.3), we obtain

$$
\begin{aligned}
& \stackrel{0}{e}_{r}=\frac{1-\sigma^{2}}{E}\left(\gamma_{2}+\delta_{2} \psi\right) \frac{\sin ^{\kappa} \psi}{r} \\
& { }_{e_{\psi}}^{0}=-\frac{(1+\sigma) \sigma}{E}\left(\gamma_{2}+\delta_{2} \psi\right) \frac{\sin ^{\kappa} \psi}{r} \\
& { }_{e}^{0} \\
& e_{r \psi}=0
\end{aligned}
$$

Remark 4.3: If $S_{1}=S_{3}=0, S_{2} \neq 0$, by virtue of (4.14), $\delta_{2}=0$ and from (4.37) there follows that

$$
v_{\psi} \equiv 0, \quad v_{r} \neq 0
$$

The last means that the points of the body under consideration displace only in the radial direction.
3. $\frac{1}{v}>\kappa>0$.

Analogously to the previous cases, taking into account that (see (4.4))

$$
(1+\kappa)(1-\kappa \sigma-\sigma)-(1-\sigma) c^{2}=0
$$

up the rigid motion we have

$$
\begin{align*}
& v_{r}=\frac{\sigma^{2}-1}{\kappa E h_{0}}\left[\gamma_{3} \cos (c \psi)+\delta_{3} \sin (c \psi)\right] r^{-\kappa}, \\
& v_{\psi}=\frac{(1+\sigma)(1-\kappa \sigma-\sigma)}{\kappa c E h_{0}}\left[\gamma_{3} \sin (c \psi)-\delta_{3} \cos (c \psi)\right] r^{-\kappa} . \tag{4.38}
\end{align*}
$$

Substituting (4.38) in (3.2), (3.3), we obtain

$$
\begin{aligned}
& \stackrel{0}{e}_{r}=\frac{1-\sigma^{2}}{E}\left[\gamma_{3} \cos (c \psi)+\delta_{3} \sin (c \psi)\right] \frac{\sin ^{\kappa} \psi}{r} \\
& \stackrel{0}{e}_{e_{\psi}}=-\frac{(1+\sigma) \sigma}{E}\left[\gamma_{3} \cos (c \psi)+\delta_{3} \sin (c \psi)\right] \frac{\sin ^{\kappa} \psi}{r} \\
& \stackrel{0}{e}_{r \psi}=0
\end{aligned}
$$

In all the above cases $(\kappa>0)$, by virtue of (4.2), after integration, from (3.4), (3.6) up to the rigid transfer we get

$$
\begin{equation*}
v_{3}=-\frac{k}{\kappa \mu h_{0}} \frac{S_{3}}{r^{k}} . \tag{4.39}
\end{equation*}
$$

Substituting (4.39) in (3.4), we obtain

$$
\stackrel{0}{e} \psi 3_{0}^{e}=0, \quad e_{e_{3 r}}^{0}=\frac{k S_{3}}{2 \mu} \frac{\sin ^{\kappa} \psi}{r} .
$$

Remark 4.4: In the particular case of a half-plane (i.e., when $\beta=0$ ), the solution of the above problem of the concentrated force is obtained in [5] (see pp. 121-129) from the solution of the problem, when the cusped prismatic shell-like body with the thickness (4.1) is arbitrarily loaded along the cusped edge $x_{2}=0$. The solution of the last problem is constructed in the integral form, preliminary solving the corresponding boundary value problem in a half-plane for the equation of the stress function. The order of this fourth order equation degenerates into the second order by $x_{2}=0$.

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