Rendiconti
Accademia Nazionale delle Scienze detta dei XL
Memorie di Matematica
101° (1983), Vol. VII, fasc. 15, pagg. 277-278

GIOVANNI PROUSE (*)

Errata-corrige to the paper « Analysis of a three-dimensional model of water circulation in a basin ».

(vol. V, fasc. 2 (1981-82))

In the above mentioned paper the equations of motion (in a weak sense) of a viscous incompressible fluid in a basin with a free surface were given by relations (4.3), (4.4), (4.5), (4.6), (4.7), (4.8).

These equations were obtained from the classical weak Navier-Stokes equations in the unknown functions $\boldsymbol{u}(x, y, z, t)$ (velocity of the fluid), $\varphi(x, y, t)$ (height of the free surface over the bottom of the basin) by means of the change of variables

$$\xi = x, \quad \eta = y, \quad \zeta = \frac{2 - \varphi(x, y, t)}{1 + \varphi(x, y, z)}, \quad \tau = t.$$

It was stated that the components u_1^*, u_2^*, u_3^* of the new unknown function

$$\boldsymbol{u}^*(\xi, \eta, \zeta, \tau) = \boldsymbol{u}(\xi, \eta, (1 + \varphi)\zeta + \varphi, \tau)$$

corresponded to the components of $\boldsymbol{u}$ in the new reference frame.

This is not so; a straightforward calculation shows, in fact, that u_i^* $(i = 1, 2, 3)$ represent the components of the vector $\boldsymbol{u}$ with respect to the system P_ξ, P_η, P_ζ, where

$$P_\xi = \boldsymbol{i}x_\xi + \boldsymbol{j}y_\xi + \boldsymbol{k}z_\xi = \boldsymbol{i} + \boldsymbol{k}(1 + \zeta)\varphi_\xi,$$
$$P_\eta = \boldsymbol{i}x_\eta + \boldsymbol{j}y_\eta + \boldsymbol{k}z_\eta = \boldsymbol{j} + \boldsymbol{k}(1 + \zeta)\varphi_\eta,$$
$$P_\zeta = \boldsymbol{i}x_\zeta + \boldsymbol{j}y_\zeta + \boldsymbol{k}z_\zeta = \boldsymbol{k}(1 + \varphi).$$

(*) Indirizzo dell'A.: Dipartimento di Matematica del Politecnico, Piazza Leonardo da Vinci, 32, 20133 Milano.

ISSN-0392-4106

Equation (4.6) must consequently be written

$$b_*(\boldsymbol{u}^*, \boldsymbol{u}^*, \boldsymbol{h}^*) = \int_{\Omega^*} (\boldsymbol{u}^* \cdot \nabla) \boldsymbol{u}^* \cdot \boldsymbol{h}^* (1 + \varphi)\, d\xi\, d\eta\, d\zeta =$$

$$= \int_{\Omega^*} \left(u_1^* \frac{\partial \boldsymbol{u}}{\partial \xi} + u_2^* \frac{\partial \boldsymbol{u}}{\partial \eta} + u_3^* \frac{\partial \boldsymbol{u}}{\partial \zeta} \right) \cdot \boldsymbol{h}^* (1 + \varphi)\, d\xi\, d\eta\, d\zeta =$$

$$= \int_{\Omega^*} \left[u_1^* \left(\frac{\partial \boldsymbol{u}}{\partial \xi} - \varphi_\xi \frac{1 + \zeta}{1 + \varphi} \frac{\partial \boldsymbol{u}^*}{\partial \zeta} \right) + u_2^* \left(\frac{\partial \boldsymbol{u}}{\partial \eta} - \varphi_\eta \frac{1 + \zeta}{1 + \varphi} \frac{\partial \boldsymbol{u}}{\partial \zeta} \right) + \right.$$

$$\left. + \left(u_1^* (1 + \zeta) \varphi_\xi + u_2^* (1 + \zeta) \varphi_\eta + u_3^* (1 + \varphi) \right) \frac{\partial \boldsymbol{u}^*}{\partial \zeta} \frac{1}{1 + \varphi} \right] \cdot \boldsymbol{h}^* (1 + \varphi)\, d\xi\, d\eta\, d\zeta =$$

$$= \int_{\Omega_{x,y}} \left(u_1 \frac{\partial \boldsymbol{u}}{\partial x} + u_2 \frac{\partial \boldsymbol{u}}{\partial y} + u_3 \frac{\partial \boldsymbol{u}}{\partial z} \right) \cdot \boldsymbol{h}\, dx\, dy\, dz = \int_{\Omega_{x,y}} (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} \cdot \boldsymbol{h}\, dx\, dy\, dz\,.$$

This different interpretation of equations (4.3) ... (4.8) does not modify the results obtained in the sequel.

We would also like to point out that the interpretation related at § 3 of the boundary condition (3.3) at the free surface must obviously be modified. Equation (3.3) imposes, as is well known, that, if at a certain time $\bar{t}$ a fluid particle is on the free surface, it remains there $\forall t > \bar{t}$.

Direttore responsabile: Prof. A. Ballio - Autoriz. Trib. di Roma n. 7269 dell'8-12-1959
« Monograf » - Via Collamarini, 5 - Bologna