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Physics as a Tool for Solving Disputes in Mathematics. A Case-Study

Through the study of the motion of projectiles, plates, pendulums, sound and light the scientific community of the 17th century become conscious of the pervasiveness of *Change*. And " the continued exploration of changing quantities (i.e. ' variables ') soon forced the scientists to realize that they required a profounder mathematical tool, and they hastened to create it " (1). It is not possible to deny this effect on the Calculus connected with physics problems, though certain key problems (e.g. the quadrature and the cubature of geometrical figures) could come from Greek geometry: mathematicians' interest here depends above all on the rediscovery of the classics of Greek science, quite independently of contemporary physics problems. The physics-mathematics relationship at the basis of the Calculus is thus complex. One aspect is the use of *physical* notions to resolve doubts, rather that the well known " application of mathematics to physical processes ".

Time and mathematics

In dealing with *variables* (i.e. quantities which change continually) certain mathematicians of the 17th century (and after) found analysis of the concept of time more useful. Time is a determinant feature of any motion or change in physics, so that any physical process runs its course in time. This is the real meaning of Aristotle's " Time is the number of motion " (2): late 17th century mathematicians saw Aristotelian concept of time: i) the only way of measuring time is by cyclical, that is, *periodic*, events, *but* ii) time in physics has to be represented by a certain mathematical scheme, which makes it into a magnitude and *qua* magnitude time is mathematically a so-called (open) linear continuum. For Bocher (3) " Aristotle's statements on time in the *Physics* are physics and *have pertinence in the physics of today* ". In all cases, this was the case for mathematicians and philosophers of Nature in the late 17th century.

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(1) M. Kline, *Mathematics and the Physical World*, New York 1959, 363.
(2) Aristotle, *Physica*, Book IV, 10-14.
(3) S. Bochner, *The Role of Mathematics in the Rise of Science*, Princeton 1966, 148, my italics.

Let us consider a convincing exemple, that of Isaac Barrow in his *Lectiones geometricae*, Lectio I:

Tempus itaque per rectam lineam semper designabimus; arbitrarie quidem initio sumptam et expositam, at caius partes proportionalibus temporis partibus, et puncta temporis instantibus respectivis juste respondebunt, et iis apposite repraesentandis inservient (4).

Velocity as a " power "

Barrow distinguisches between *change* and *rate of change*. We stress the importance of his definition (in *Lectiones geometricae*, 1670) of Velocity as " a power ":

definitur idcirco velocitas potentia, quâ mobile spatium aliquod in aliquo tempore pertransire potest (5).

Furthermore, dealing with time-magnitude as a linear continuum, Barrow gives his version of the *labyrinthus continui*, and he criticizes theses traditionally attributed to atomism (" the continuum consists of *indivisible* parts "). According to the Lectio I:

Omnis temporis instanti, seu indefinite pervae temporis particulae; (instanti dico, vel indefinitae particulae, nam uti nihil adomodul refert, utrum lineam ex innumeris punctis, and ex indefinite parvis lineolis compositam intelligamus; ita perinde est, utrum tempus ex instantibus, an ex innumeris minutis tempusclusis confratum supponamus; nos aaltem brevitati consulentes pro temporibus quantumlibet exiguis instantia, hoc est pro tempuscula repraesentantibus lineolis puncta non verebimur usurpare); cuilibet dico temporis momento competit velocitas aliquis gradus, quem mobilem tunc habere concipiendum est (6).

We personally feel this to be a kind of typical " reduction " from *partes indivisibiles* to *partes semper divisibiles*, yet too small (*indefinite parvae*) as the geometrician's " instinct " is to subdivide them, in accordance with the Continentals' use of *indivisibilia* (Cavalieri, Torricelli, etc.) in quadrature maths. As shown in Giorello, Perucca (7), this was typical in Europe in " conciliating " mathematical *indivisibilia* with Aristotle's (physicall) concept regarding time (and space).

A part form this weakly nominlist idea of temporal and spatial *indivisibilia*, Barrow adds a strongly nominalist idea, e.g. in his *Lectiones mathematicae*:

Quin et eodem modo tempus ex temporibus, non ex instantibus; motus ex motibus, non ex tendentiis indivisibilibus; velocitas ex velocitatibus, et

(4) W. Whewell (ed.), *The Mathematical Works of Isaac Barrow*, Cambridge 1860, 166.
(5) *Ibidem*, 167.
(6) *Ibidem*, 167-168.
(7) G. Giorello, M. Perucca, " *Strain of Imagination*: una nota su metafisica e scoperta matematica ", *Riv. Stor. Sci.*, 1 (2), 1984, 279-305.

pondus ex ponderibus, neutram ex grandibus vel impetibus absolute minimis; numerus ex unitatibus, illae ex fractis partibus, non ex cyphris, constant et integrantur (8).

In Barrow's opinion, we cannot conceive infinite division, but must accept its possibility. *Indivisibile* is a *plane nihil*.

Barrow insists strongly on the conciliation between the new Calculus and Aristotle's tenets, also as this corresponds to his own mathematics. In Lectio X of the *Lectiones geometricae*, advised by Newton, he includes algorithmic rules for tangents; he uses e and a for the *indefinite parva* increments in the independent variable x and the dependent variable m.

1. Inter computandum omnes abicio terminos, in quibus ipsarum a, vel e, potestas habetur, vel in quibus ispae ducuntur in se (etenim isti termini nihil valebunt).
2. Post aequationem consitutam, omnes abicio terminos, literis constantes quantitates notas, seu determinatas designantibus; aut in quibus non habentur a, vel e; (etenim illi termini semper, ad unam aequationis partem adducti, nihilum adaequabunt).
3. Pro a ipsam m; pro e ipsam t substituo (9).

Barrows adds to these rules (which can be applied in the case of algebraic equations):

Quod si calculum ingrediatur curvae cujuspiam indefinita particula; substituatur ejus loco tangentis particula rite sumpta; vel ei quaevis (ob indefinitam curvae parvitatem) aequipollens recta (10);

thus illustrating a technique basic to all the constructions of trascendental curve tangents (11).

Barrow's method has been interpreted as a particular case of the approach *via* moments (Newton) or the approach *via* differentials (Leibniz) to the geometrical or mechanical problems. In William Whewell's 1860 Barrow edition, the Latin of the *Regula prima* (*etenim isti termini nihil valebunt*) is translated (in Whewell's Preface) as: " for they are of no value compared with the rest, as being infinitely small " (12).

But this is after 1850: after the Cambridge analytical school's revaluation of the *differential* Calculus and after de Morgan's comments on the history of the Calculus. In fact Whewell believes that " the substantian identity of Barrow's and Leibniz's methods is evident " (13). A fine exemple of how to create precursors!

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(8) W. Whewell (ed.), *The Mathematical...*, *quot.*, 139.
(9) *Ibidem*, 247.
(10) *Ibidem*.
(11) Cf. E. Giusti, " A tre secoli dal calcolo: la questione delle origini ", *Bollettino UMI*, (6) 3-A, 1984, 1-55.
(12) W. Whewell (ed.), *The Mathematical...*, *quot.*, XIII.
(13) *Ibidem*.

However Barrow was *not* using Leibniz language (as Whewell would have it), but the words *etenim isti termini nihil valebunt* could have a different meaning to Barrows contemporaries.

In his *Considerations circa analyseos ad quantitates infinite parvas applicatae principia* the Ducth Theologician Bernhard Nieuwentijdt (1694, Amsterdam) follows a different interpretation of Barrow's (and Leibniz's or Newton's) procedures. In calculation such as the differentiation of $y = x^2$ (or, in general $y = x^n$) infinitesimal products such as $dx \cdot dx$ (or, as Barrow would have it: $e \cdot e$) should be equal zero, because the product of two quantities lower that " any assignable quantity " can only be nothing. (For Nieuwentijdt, indeed, we can " enlarge " the linear continuum R with nilpotent entities: algebraically, we embed the field of real numbers R in a ring R' with zero divisors. For Nieuwentijdt this is the only valid way of justifying Barrow's algorithm rules).

There is no room here for the European disputes caused by Nieuwentijdt's attitude (with replies from Johann Bernoulli, Jacob Hermann and Leibnihimself, with his emphasis on his " metaphysical " *loi de continuité*). In Nieuwenz tijdt we find certain religious scruples (14) and a different concept of algebraic structure underlying that type of algorithm. We find also in *Considerationes* (and in other Nieuwentijdt's mathematical works) a particular kind of " atomism " (15). This " atomism " is not only in Nieuwentijdt's justification of current ideas in the Calculus (such as: the circle is an infinetly sided polygon) but also in his refusal to consider Leibniz' second differentials meaningful. If dx and dy, etc. (or Barrow's e and a) are *atoms* (literally indivisible!), how is possible to subdivide them infinitely, passing on to ddx and ddy, etc.? Nieuwentijdt felt that Leibniz' notation was misleading just where we see its merits it suggests the iteration of d as an operator (16)), and pretended to keep to Barrow's notation.

Nieuwentijdt's position was not calculated to win success among his contemporary mathematicians. These English mathematicians must have been perplexed when faced with these theses, when they realised that, at least on inspection, the theses were against the " endless " idea od " rate of change of rate of change ", so central to Newton's approach to mechanics. Nieuwentijdt's opinions even stimulated polemical mention by George Berkeley in his fragment " On Infinities " (17).

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(14) P. Vermeulen, " Nieuwentijdt's Criticism of the Leibnizian Calculus ", to appear.

(15) Cf. G. B. Boyer, *The History of the Calculus*, New York 1939, 214. We agree with Boyer wout the " static " nature of infinitesimal in Nieuwentijdt's *Considerationes*; we do not agree with this author that Nieuwentijdt's justification of differential algorithm is only " a less critical manipulation " of infinitesimal quantities than Leibniz: the real difference in *a change of the underlying algebraic structure*.

(16) Cf., on this subject, H. J. M. Bos, " Differentials, higher-order differentials and the derivative in the Leibnizian calculus ", *Arch. Hist. Exact Sci.*, *14*, 1974, 1-90.

(17) Cf. also the Bishop Berkeley's criticisms of higher order differentials in G. Berkeley, *The Analyst*, London 1734, section 5.

Berkeley's main target in the *Analyst* (1734) was Newton's Calculus, especially Newton's novelty about " calculus of Infinities ". The novelty of Newton's playing with infinity, however, was well pointed out by Hoené Wronski in his *Philosophie mathématique*: we do not find in Fermat and other mathematicians committed with *indivisibilia* the " attention aux fonctions algorithmiques ", which constitue the hard core of Newton's and Leibniz' approach to Analysis (18).

The same must also be said for Newton's " debt " to Barrow; we think along D. T. Whiteside (19) and differently from Child (20), that this " debt " is not to be over-estimated.

In effect, Newton went far beyond Barrow's concept of " *indefinite* parvae partes " of a continuum (time, space, etc.) (21); his " playing " with higher order differentials (fluxions of fluxions, etc., more exactly) superseded the ideas of physical atomism and mathematical *indivisibilia* completely.

In his *Janus* article (22) about Berkeley's compensation of errors, Ivor Grattan-Guinness has pointed out that also Berkeley's criticism of Newton has its roots in some kind of atomism, which presupposes the atomistic perception of *minima sensibilia*.

It was Berkeley's criticism in *The Analyst* which stimulated Colin Mac Laurin to reply to Berkeley in his *Treatise of Fluxions* (1742): we find here the reference to physical grounds to justify Newton's algorithmic " playing with infinity ".

In the first Chapter of Book I of the *Treatise* Colin Mac Laurin uderlines mathematics as the science of relationships (23); on p. 52 he takes up a thesis, which was dear to Barrow, that the objects to which primitive terms of a mathematical theory refer, need not necessarily esist in the real world, providing that we have a clear idea of their " relationship ". On p. 53 we have the concept of time as an open linear continuum, which, given the local and limited nature of our experience, we are obliged to " close ". This kind of time is obviously a measure of change (like in Barrow, and in Aristotle); but as the substrate of the change is space, the *Velocity* will have to be considered as a kind of primitive term, assumed as something for which to have a precise numerical determination *via* physical considerations (Barrow's *sensus communis* comes to mind).

Following Barrow, Mac Laurin told about Velocity as a power, and

This being evident, it does not seem to be necessary, in pure geometry, to enquire further what is the nature of this power, affection, or mode, which

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(18) H. Wronski, *Philosophie mathématique*, citrà 1900, 57-58.
(19) See D. T. Whiteside, " Isaac Barrow ", in G. C. Gillispie (ed.), *Dictionary of Scientific Biography*, 14 vols., New York 1972-1976 vol. 1, 473-476.
(20) I. Barrow, *Lectiones geometricae*, Engl. trans.
(21) See, for example, Newton's change from *indefinite* to *infinite* in D. T. Whiteside (ed.), *Newton's Mathematical Papers*, Cambridge 1900, vol. III, 80.
(22) I. Grattan-Guinness, " Titolo ", *Janus*, —
(23) C. Mac Laurin, *Treatise on Fluxions*, Edinburgh 1742, 51.

is called *Velocity*, and is commonly ascribed to the body that is supposed to move. It seems to be sufficient for our purpose that, while a body is supposed in motion, it must be conceived to have some velocity or other at any *term* of the *time* (*my italics*) during which it moves (24).

The idea of *Velocity* will have to be left to the physicists; for the mathematician it is sufficient if every given *term of time* corresponds to a velocity for the object in motion.

On the following page, however, Mac Laurin quotes Barrow's idea of Velocity " as a power " more accurately. But Mac Laurin also assume the task of *justifying* a calculus with " velocity of velocity " (" fluxion of fluxion ") and so on; our question is now: in a Barrow–Aristotelian framework, is there anything conceivable like *a power of a power*? In our opinion, the answer is *No*.

To get round this, however, Mac Laurin proposes a meaning shift (for " Velocity "), by defining velocity as " the action of a power ", thus taking up one of Newton's key ideas, which is well beyond both, the implicit atomism of the mathematical *indivisibilia* and Barrow's simple idea of *partes indefinite turvae* (25).

(24) *Ibidem*, 53.
(25) H. J. M. Bos, " Differentials... ", *quot.*, has pointed out how there was a similarly radical change in the Continent with Leibniz and the Bernoullis.