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History of Electrodynamics: W. Weber and R. Clausius on the Principle of Conservation of Energy

There were two main metaphysical roots to the mechanical principle of conservation of energy (PCE). Both were connected with the principle of impossibility of perpetual motion. The first one, summarized in the statement: « ex nihilo nil fieri », asserted the impossibility of obtaining an infinite quantity of work in a cyclical path under the action of a given force. The second one, summarized in the statement: "*nil fieri ad nihilum*", asserted the impossibility to destroy a certain quantity of work. While the first root was connected with the work–vis viva thorem and the birth of the mathematical concept of potential, the second one was more attached to the idea of a non vanishing causal relation (*causa aequat effectum*) and to the definition of a physical quantity that might account for this requirement. But the history of the PCE debate is not confined to mechanics. The two same roots of the principle had an influence on the Classical Electromagnetic Theory (CET) debate. The contiguous action approach was related to the physical idea of a « substantial » energy localized in space and moving following a continuity equation (*nil fieri ad nihilum*). The action–at–a–distance approach was intead related to the work–vis viva theorem and to the mathematical concept of potential (*ex nihilo nil fieri*).

Both ideas of energy did not deserve much attention by the secondary literature. This paper aims at an analysis of two interpretations of PCE in electrodynamics: one by Weber and one by Clausius. Both were related to a force law of action–at–a–distance. But both force laws at variance with Coulomb's one depended on velocities and accelerations. It is thus interesting to analyse in which way conservation of energy was claimed.

W. Weber (1804–1891) was the main scientist of the elctrodynamic action–at–a–distance approach. His contributions were fundamental, both for CET and for PCE. As far as CET is concerned, W. Weber in 1846 gave a force law for the interactions between electric particles which, while still following the action–at–a–distance with infinite speed approach of Newton, Coulomb and Ampère, substantially modified previous results. In fact, Weber considered that forces depend not only on distance but also on velocities and accelerations. This new interpretation of the concept of force gave rise to a debate which is

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still alive today. Another remarkable aspect of Weber's approach is the use of relative and not absolute velocities, that is, his lack of use of a privileged reference frame. In fact, accepting the basic assumptions of G. T. Fechner, his predecessor in the chair of physics in Leipzig, Weber developed an atomistic conception of electricity and assumed that electrical currents were electrical particles of opposite sign in opposite motion. The particles were assumed to possess electrical inertia. The force law which he derived, and the related potential, were partly modelled on Newtonian gravitational attraction in that central forces were assumed. The main changes in this theory were the dependence of forces not only on position but also on velocity and acceleration (two previous contributions in this direction were given by Gauss in 1835 and Riemann in 1861).

The expression of the law is:

$$F = \frac{e_1 e_2}{r^2}\left[1 - \frac{1}{c_w^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{c_w^2}\frac{d^2r}{dt^2}\right]$$

in electrostatic units, where dr/dt is the relative radial velocity and d^2r/dt^2 the relative radial acceleration and c_w a constant expressing the ratio between the electromagnetic and electrostatic unit of charge. This law was published in the first of Weber's famous "Elektrodynamische Maassbestimmungen", seven long works brought out between 1846 and 1878. This law, even more elementary than Ampère's, can explain the lack of electrostatic effects in a current of uniform density. It is reducible to Coulomb's law in the static case, and can explain not only Ampère's forces between currents but also Faraday's phenomena of electromagnetic induction and Neumann's mutual potential.

It immediately appeared in 1847 that Helmholtz's PCE was in direct conflict with Weber's law for electrodynamics. This was in fact not a central force law depending only on distance but a law of forces depending on velocities and accelerations. PCE rapidly acquired great importance in the community of physicists and for Weber the disagreement of his law with the PCE of Helmholtz (based on Newtonian forces) was a serious problem, despite the fact that his law provided a good account of all the experimental results of the time. This is the starting point of a very long debate between Helmholtz and Weber. Weber in 1848 drew attention to an extraordinarily important result: his force law could be derived from the potential

$$V = \frac{ee'}{r}\left(\frac{1}{cc}\cdot\frac{dr^2}{dt^2} - 1\right)$$

Of course the force being dependent on velocities and accelerations, the potential cannot be a function of the coordinates only, but will depend on the velocities. This potential is thus greatly different from both kinetic and poten-

tial energy of Helmholtz. The distinction itself between potential and kinetic energy is now under discussion: from a Weberian point of view they do not appear as basic entities any more, but only as a specific choice connected with the Newtonian model of central forces depending on distances. Another aspect of the problem is the following: does Weber's force obey another PCE if it does not obey the PCE of Helmholtz?

Since Weber's force is derived from a potential, it is in agreement with the above recalled first root of PCE: "*ex nihilo nil fieri*". In fact the work done by the force is a complete differential, and in a closed trajectory there is no production of work: there is no disagreement with the impossibility of perpetual motion. The only novelty is that now the vis viva will depend not only on positions but also on the velocities at these positions and by a closed path it has to be meant that the final point must have the same position and the same velocity as the initial one.

W. Weber's main contributions to electrodynamic theory were published from 1846 to 1878. During this time Weber had a lengthy polemic with Helmholtz and Clausius, who both contested Weber's law. But this opposition to Weber's law had two different roots. On the one hand its relations with PCE, on the other, the doubtful assumption of oppositely charged particles moving with opposite relative velocities. The latter problem will be discussed in the section on Clausius, being largely the object of the controversy with Clausius. The former problem was the main object of the controversy with Helmholtz.

This controversy is extremely relevant for my main thesis. Indeed it demonstrates that PCE was a principal basis for the CET debate; that the two mechanical roots of the principle were still connected to different interpretations of PCE and also shows Weber's sophisticated interpretation of PCE. The long controversy can be divided into two parts: a) from 1847 to 1870 and b) from 1870 to 1898. The first part referred to the compatibility of Weber's law with PCE; the second part to the possibility of deriving absurd consequences from Weber's PCE.

Clausius' main contributions to PCE and CET were published in the seventies. His approach is linked with the "*ex nihilo nil fieri*" aspect of the impossibility of perpetual motion, that is, with a potential law. He is not so much interested in the heuristic of energy as an indestructible quantity assuming different forms, nor in Leibnizian metaphysics, nor in the Newtonian model, but in the justificative aspect of PCE; the force law has to be compatible with PCE. PCE is interpreted as the requirement that the work in the work–vis viva theorem has to be a complete differential. Thus there is no restriction in the form assumed by the forces: they do not have to be central, and can depend on velocity and accelerations. Nor is the clear distinction between potential and kinetic energy relevant: all that matters is that the work done by the assumed forces is a complete differential. Clausius himself gave a force law depending on velocities and accelerations and his analytical approach had important consequences.

This analytical approach was already evident in Clausius' earlier works in the fifties, which were at variance with Helmholtz's own development which began to be more analytical in the great paper of 1870. In 1852, in a paper on the mechanical effects and the heat generated by an electric discharge and by an electric stationary current, Clausius introduces his main line of thought. The link with the mechanical tradition is explicit: " In the following paper I have endeavoured to reduce the effect produced by an electric discharge to a definite unit derived from the principles of mechanics [...] ".

The relevance of this approach consists in the fact that if a difference of electric potential satisfies the work-vis viva theorem, the only condition on electrical forces is that they admit a potential (in modern terms a work function), *i.e.* that their work is a complete differential. This is Clausius' interpretation of PCE. No reference to conservation of a specific quantity is mentioned, nor specific models of forces assumed.

This same approach was extended in 1859 in a textbook on *Potential Function and Potential*, where again the problem is discussed in its analytical generality and the restriction to the particular case of the Newtonian forces is always explicitly considered due to the special interest of these forces for physics. In the second part of the book the derivation of the work-kinetic energy theorem is made from the Principle of Virtual Velocities and from d'Alembert's principle. The variational approach is thus given priority over PCE, in agreement with modern formulation, and reverts the priority established by Helmholtz. The agreement consists in the fact that in contemporary views the force function (work function in modern terminology) and not the potential energy is considered the basic quantity of analytical mechanics. The specific cases of central forces, leading to the standard concept of potential energy, are always seen by Clausius as particular restrictions under which the function representing the work can easily be integrated. In 1876, in the first book of the second edition of his *Mechanical Theory of Heat*, Clausius refers back to this problem and specifies that in the case in which the expression of the work done during an indefinitely small motion is a complete differential of a function of x, y, z, this function « plays a very important part in our calculations ». In fact:

Hamilton gave to it the special name of " force function "; a name applicable also to the more general case where, instead of a single moving point, any number of such points are considered, and where the condition is fulfilled that the work done depends only on the position of the points. In the later and more extended investigations with regard to the quantities which are expressed by this function, it has become needful to introduce a special name for the negative value of the function, or in other words, for the quantity, the subtraction of which gives the work performed; and Rankine proposed for this the term " potential energy ". This name sets forth very clearly the character of the quantity; but it is somewhat long, and the author has ventured to propose in its place the term " Ergal ".

Clausius' choice of the term did not have great success, but it is important to note the generality he attributes to its meaning: " Among the cases in which the force acting on a point has an ergal, the most prominent is [...] the case in which the force may be classed as a central force ". And again:

The assumption lying at the root of the foregoing analysis, *viz.* that central forces are the only ones acting, is of course only one among alla the assumptions mathematically possible as to the forces; but it forms a case of peculiar importance, inasmuch as all the forces which occur in nature may apparently be classed as central forces.

No better expression could be found of Clausius' approach: the mathematical structure gives many possibilites, some of which are physically interesting. This point of view has one merit and one demerit relative to Helmholtz's. The merit lies in its greater generality. In the previous year Clausius introduced an electrodynamic force depending on velocities, giving up the 'apparent' restriction to central forces, and his analytical approach to the work-vis viva theorem was not to require modifications. In Helmholtz's PCE on the other hand, the central forces played a basic, not easily modifiable, role. The demerit lies in the lack of heuristic power: Clausius' electrodynamic potential was derived from Helmholtz's approach and was Clausius' only contribution to CET. Finally, Clausius formulates a first version of PCE in which he says that if the forces acting on the system have an ergal (*i.e.* if the work is a perfect differential of some function of the coordinates), the sum of the vis viva and of the ergal remains constant during motion. This expression of PCE was to be reprinted in exactly the same way in the 1887 edition of the book. The important requirement of Clausius' PCE is that the work has to be a *perfect differential* and not that the forces have to be central. This very " modern " approach was to find an application in a contribution given to CET by Clausius himself. In 1875 in fact, in a very short paper he expresses a basic law of electromagnetism, and in 1876, in a second short paper, analyses the relations of this law with his own PCE. Clausius' law depends on a potential which he derives from F. Neumann through the reformulation of Helmholtz. But instead of the current elements, Clausius utilizes charges in their velocities. The main feature of Clausius' force law is its dependence on velocities and accelerations, while at the same time it is different from Weber's: " [...] for reasons quite independent of Helmholtz's, the conviction has forced itself upon me that it (Weber's law) does not correspond to reality ".

The differences between Clausius and Weber are: a) Clausius did not accept Fechner's hypothesis of two opposite charges moving with opposite velocities; b) Clausius considered absolute and not relative velocities; *i.e.* he introduced a privileged reference-frame. (Helmholtz's reasons for criticising Weber were connected with the interpretation of PCE).

Referring to his own formula, Clausius does not as yet give the " reasons which have induced me to advance it ", but just two remarks " of elucidation ":

the first refers to the fact that assuming an electrodynamic action that " takes place through an intervening substance ", the dependence cannot only be upon the relative motion of the particles but also on their absolute motion with respect to the medium. Second, in this case, the action between the particles does not necessarily have to be in the line connecting the two particles; that is, central forces are superseded.

Clausius' approach was to be corroborated by later results. The great importance of Clausius' law for subsequent research is emphasised by Whittaker. According to Whittaker, Lorentz's theory is " a combination of Clausius' theory of electricity and Maxwell's theory of the aether ". The only change which is necessary to make in Clausius' theory is that of retarding the potentials in the way indicated by L. Lorenz. With these modifications, Clausius' theory was to give an essential contribution to Lorentz's theory, *i.e.* the kinetic potential from which a Lagrangian derivation gives Lorentz's force.

Furthermore, Clausius' kinetic potential is one step towards the derivation of the complete set of equations for the electromagnetic field, including the ponderomotive equation (Lorentz's force), from a Principle of Least Action (PLA). This was to be derived by Schwarzschild in 1903.

Summarizing the results of Weber's and Clausius' electrodynamic papers it can be said that they made basic contributions to the field. Both of them provided force's laws in substantial agreement with most experimental results. Whatsmore they produced expressions of the PCE which agreed with their own force laws. Thus the " *ex nihilo nil fieri* " tradition showed a great justificatory power. But the real difference with the contiguous action school relies in my view in the heuristic power of the two approaches and mostly in the heuristic power of the specific PCE adopted. After 1885 (Poynting) there were few doubts that a contiguous propagation PCE was " simpler " than an action-at-a-distance one. The localization of energy allowed the separation of the globality of the universe (needed for the definition of potential energy) in small parts. The energy of these parts was simply defined by the consideration of their volume and surface. But to understand the sofistication of the debate, its non experimental solution, the heuristic confrontration of different version of PCE, the acknowledgement of the justificatory merits of Weber's and Clausius' PCE is a prerequisite.

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