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Beniamino Segre: his life, his work (**)

Beniamino Segre was born in Turin on February 16, 1903. Upon completion of his secondary school studies, at only sixteen years of age, he won a scholarship from the Collegio Carlo Alberto to continue his studies in mathematics at the University of Turin.

At the University he had the good fortune to have famous mathematicians as his teachers: Giuseppe Peano, Gino Fano, Guido Fubini, Carlo Somigliana and Corrado Segre. The latter was a relative of his on his mother's side: Beniamino's mother (she too was of the Segre family) was a first cousin to Corrado Segre.

It was quite natural therefore that the young Beniamino chose his cousin Corrado as his advisor for his thesis. Corrado was 40 years older, and was considered internationally one of the greatest representatives of the flourishing Italian school of algebraic geometry, teacher of several already well known algegraic geometers, including among others the great mathematician Francesco Severi.

Moreover, it was certainly not only the relationship which attracted the student Beniamino to Corrado Segre. In fact, he was to write later of his cousin-teacher: "His lecture was the high point of the day, when, inspired by the sacred fire of science, he exerted a great charm over his pupils ... I had the privilege of following his courses in 1921-22 and 1922-23, of taking my degree under him and of having frequent contact with him... The spendid example of his life and the memory of his exceptional gifts of mind and heart

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were a precious *vade mecum* for me" [1].

B. Segre graduated at only 20 years of age, with a brilliant thesis in algebraic geometry, the essential points of which were promptly published in the "Atti dell'Accademia delle Scienze di Torino". As often happens, the path on which he embarked in preparing his thesis was to be decisive for the outstanding development of the personality of the future scientist; though it must be said immediately that the fields of research in which Segre was to leave his mark include many branches of mathematics, and of its application.

The following year Corrado Segre died suddenly (at the age of only 61 years), and so Beniamino became assistant in the University of Turin, for three years, to the chairs of Rational Mechanics and of Analytic, Projective and Descriptive Geometry. His eager mind prompted him towards the most varied and interesting researches in algebraic projective and differential geometry, etc., which led to publications as varied.

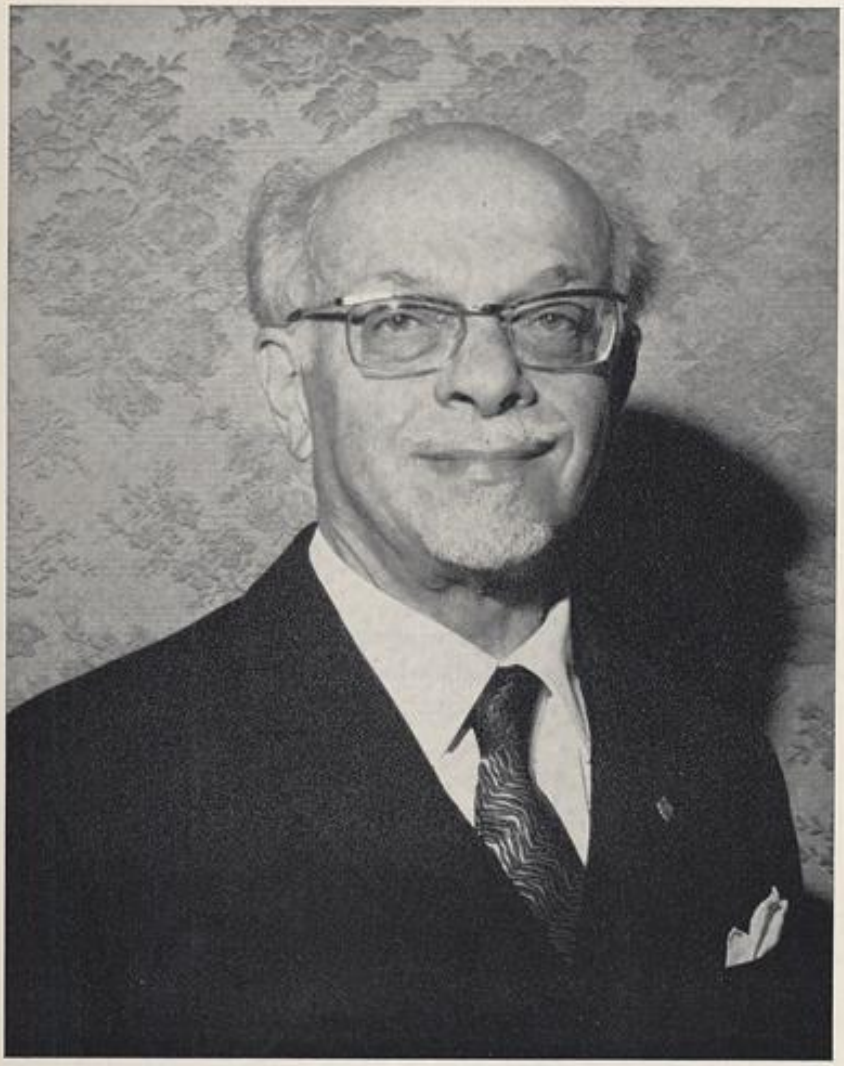
Then he spent a year in Paris with a Rockefeller scholarship, working under the guidance of the great French mathematician Élie Cartan, master of algebra, analysis and geometry. This was a very formative year for the young Beniamino, who in fact was subsequently to manifest, for every question which he undertook to treat, a secure mastery of the most diverse and complex mathematical means. And the fact is not without significance that he was offered a position as professor of geometry at the University of Zurich as an alternative to the Paris fellowship. But scientific interest prevailed over ambition.

A still more important factor in his mathematical formation was the subsequent four years during which he was assistant to Francesco Severi in Rome. The most famous pupil of Corrado Segre had summoned to his side the young cousin of the master who had died so prematurely. He could not have made a better choice! During those four years, in fact, B. Segre published 30 papers on algebraic geometry, differential and algebraic-projective geometry, on differential equations, on analytic functions of several complex variables, on topology and other topics. And this is the rhythm that will continue throughout his life.

We are now in 1931: Segre, 28 years old, wins the competition for the chair of Analytical and Projective Geometry at the University of Bologna. He teaches this subject and "Geometria superiore" until 1938, at which time he is dismissed by the application of the monstrous racial laws. But by then the name of Beniamino Segre is well known both in Italy and abroad, so that, upon the initiative of the Society for the Protection of Science and Learning, he is called to England, where he teaches at the Universities of London, Cambridge and Manchester.

This English period was filled with experience and new scientific interests, but also with sad events.

[1] Translated from: B. Segre, *Nel primo centenario della nascita di Corrado Segre*, « Rend. Sem. Mat. Univ. Torino », Vol. 23 (1963-1964).



B. Segre had left Italy with his young and courageous wife and three young children. But as soon as Italy entered the war he was separated from his family for two months. Later he was deeply grieved by the loss of his youngest child, Ornella, who was only 3 years old. These events and many other lesser difficulties certainly strengthened the character of Beniamino and his beloved Fernanda, who continued to accompany him with serenity and simplicity until she died quite suddenly, a year before his own death.

At the end of the war, in 1946, Segre was finally able to return to Bologna. A few years later, in 1950, he was called to Rome to the chair of "Geometria superiore", a chair which had until then been held by the most famous Italian geometers: L. Cremona, G. Castelnuovo, F. Enriques, F. Severi. The latter at that time became president of the "Istituto nazionale di alta matematica", and professor of "Alta geometria" of that Institute.

B. Segre succeeded F. Severi also in the chair of "Alta geometria", which he occupied for thirteen years. During these years Segre's courses, profound and perfectly organized, were attended with the greatest interest by numerous young and not so young researchers, among whom was Francesco Severi himself.

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Beniamino Segre was a mathematician of the highest order, universally recognized as such, in Italy and abroad, although his somewhat gruff nature and his steady and complete involvement in his research activity did not permit him to cultivate social relationships extensively.

His work is exceptionally copious (more than 400 publications, of which a dozen or so are entire volumes) and at the same time varied inasmuch as it penetrates into many fields of mathematics and sometimes into other fields as well.

It is certainly not possible here to give a complete account of the work of Segre, for that would require too much time and too much patience on the part of the listeners. Indeed I am quite aware that there is no true mathematics without effort and suffering. However, I hope I may be excused if (although in general and very brief terms) I speak a little more at length of the field in which Segre made the most outstanding contributions, many of them indeed fundamental. I believe that only in this manner those who are not mathematicians can appreciate the value of the scientist whom we honour today.

The field to which I refer is that of "Classic Algebraic Geometry" and, in particular, of so-called "Algebraic Geometry of the Italian school". Algebraic geometry studies the objects that can be represented by algebraic equations, that is, equations very simple at first sight (these are studied also in elementary

algebra in their simplest forms). But such simplicity hides "mathematical phenomena" which are extremely profound.

I use the term "phenomena" which is more proper to physics, because the objects of algebraic geometry, as F. Enriques used to say, are in some way analogous to those of physics. In fact they are already completely determined by the simple written equations (or rather thought equations - for they are generally not so simple!). Nor is it possible to limit or approximate these equations without irremediably falsifying their global content, which is precisely what is interesting about them.

But why "geometry", even though it be with the adjective "algebraic"? A rough reply can be given promptly. The simplest and most expressive way to study algebraic equations is to consider their solutions and to represent such solutions by points in a convenient space. If the solutions are finite in number, we get a finite group of points, but if they are infinite in number we get a curve, a surface, etc., and in the general case we get what is called an "algebraic variety of dimension 0, 1, 2, ..., k". This point of view has been very far advanced, particularly by the Italian school, whose characteristic is precisely that of using the geometric spirit in order to obtain, in the simplest and most intuitive way, profound results.

There is still another preliminary consideration. Algebraic geometry does not study curves, surfaces and algebraic varieties as they appear, defined by a system of algebraic equations. This is the usually more elementary task, that belongs to the field of projective geometry. Instead, algebraic geometry considers an algebraic variety as a "model" of a more general "algebraic object" which in fact is identified with the infinite class of models, "equivalent" among themselves, that is, they are such that we can pass from one model to another by a bi-univocal (one-to-one) algebraic transformation, that is "birational" (birational transformations are the simplest transformations in the algebraic field, yet they can even change the dimension of surrounding space to which two equivalent models belong).

So, from this point of view, the interesting properties are those which are the same for the various models of the same algebraic object, and thus, as we say briefly, they are "invariant" properties of an algebraic variety.

Among these properties those of greatest importance are the "canonical properties", that is, those properties which are univocally determined by each algebraic object, so that they characterize the object or at least they help to characterize it.

For example, regarding the algebraic curves, an invariant property is the so-called "linear series of groups of points" (a group of points is an algebraic variety of zero dimension, a linear series of these groups is an infinity of groups belonging to the algebraic curve and depending linearly on one or more parameters). Among these linear series a well known one was the "canonical

series" on an algebraic curve, and, analogously, on a variety of k dimension, the canonical linear system of $k-1$ dimension was known. That is, in each case, it was a matter of a dimension of a unity lower than the dimension of the surrounding variety. But, for $k = 2$, that is in the case of algebraic surfaces, F. Severi had encountered in 1932 an extraordinary novelty, a canonical series of groups of points on a surface, and that is, something which corresponded to a decrease of two units with regard to the dimensions of the surface. And it was in fact this discovery that induced Severi to formulate at once the general lines of the theory (revolutionary for that time) of series and systems called "of rational equivalence", consisting of varieties of dimension lying between zero and $k-1$, on an algebraic variety of dimension k (but not, it must be noted, of "canonical" series and systems of equal dimensions).

A few years later, from 1934 to 1936, after some doubts, the conviction spread that much was to be expected from the new ideas introduced by Severi. And indeed various facts, though not yet coordinated, encouraged this hope.

I have already said that the properties of algebraic geometry often have a quality which makes them resemble phisical phenomena. So there occurs to my mind an analogy with physics (presumptuous but expressive, and the physicists, I hope, will forgive me). The situation of the algebraic-geometer of that time resembles that of the theoretical physicist of today facing the various phenomena of the elementary particles: it is a question of finding a synthesis, indeed very deeply hidden, in which all these phenomena fit.

To come back to the matter of algebraic geometry and the work of B. Segre, it was a question then of finding ideas and methods which could create harmony and light where there had been only some chords and a glimmer of light here and there (I shall mention only the names of A. Comessatti, M. Eger and J. A. Todd among the precursors). Beniamino Segre accomplished this remarkable task in four basic papers (which together would constitute a volume of some 350 pages).

The first two papers, which appeared in 1934 and 1936, are preparatory. The second, which delves deeper into the theory of the tridimensional algebraic varieties as a first experiment, was awarded a prize by the Royal Academy of Belgium. The third paper and the most complete and profound entitled, "Nuovi metodi e risultati nella geometria sulle varietà algebriche", came out only in 1953, followed the next year by the fourth paper, which includes some important additions to the third one.

Thus, between the first two and the last two papers there is an interval of 17 years, although Segre had already comunicated the essential results of the third paper in a lecture given by him at the University of Cambridge in 1939. The long interval can be explained, only in part, by the difficulties involved in the transfer of the Segre family to England, which have already been referred to. I think, however, that the principal reason for the delay is found in Segre's wish to be sure of the exactness of his arduous work by going more deeply into each

aspect and each implication of his theories. These were as new in method as they were profound in their ideas, and he was developing and clarifying them "in small doses", while publishing numerous other works on topics not only in the field of algebraic geometry but also in other fields.

Indeed it is not possible to speak in just a few words of the content of the third paper, which was outstanding also for its precision and clarity. Permit me, however, to attempt to talk briefly about two basic ideas.

Consider for a moment an algebraic variety immersed in another one of a larger dimension (for example, a curve contained in a surface). Segre associates the immersed variety with what he calls "covariant succession of immersion". It is a succession of subvarieties of the immersed variety, a succession which, in general, is not easily defined but which proves to be of basic importance. I should like to say only this, that two successions of this type can be multiplied by each other as numbers or rather as polynomials, and an "inverse" of such a succession can be constructed.

This last operation, essentially of a formal algebraic nature, is a basic key. In fact it leads us to define, as Segre does, for every algebraic variety of k dimension, canonical subvarieties of dimension between zero and $k-1$. The definition is simple though hidden. Approximately one can say this: let us think (and this is always possible) of a given algebraic variety as if it were a "diagonal" variety immersed in the product of the variety itself and of a copy of it. Such immersion then gives rise to a "covariant" succession of immersion according to Segre, which is here "invariant" since the product variety is now singled out by the considered variety. Thus, the inverse of the mentioned succession gives the canonical varieties of the various dimensions contained in the original variety.

The importance of this concealed definition of the canonical varieties lies also in the fact, established by Segre, that from such a definition it follows that the canonical varieties have a topological-differential nature, that is, they are independent of the more complex algebraic nature of the varieties under consideration, with consequences of very great importance. The English mathematician D. B. Scott, on the occasion of conferring on Segre the Doctorate *honoris causa* of the University of Sussex a year and a half ago, expressed this opinion regarding the results mentioned: "The repercussions of this work, from which the whole apparatus of Characteristic Classes arose, are still reverberating through many fields of mathematics".

I, for my part, would say that, if Beniamino Segre's contribution to algebraic geometry had been limited to the four papers mentioned, his name, for these alone, should be placed among those of the greatest algebraic geometers of the Italian school.

But the work of Segre in algebraic geometry is not the only work to which we have referred. I feel I must mention at least the principal subjects to which he has made contributions of great importance, and that is: the theory of modules

of algebraic curves and surfaces, multiple planes and branching curves, integral Cremona transformations, special algebraic varieties, among which the non-singular cubic surfaces (to which Segre dedicated a fascinating volume during his stay in England), problems of intersection and of resolution of singularities of algebraic varieties, and finally, the foundations of algebraic geometry, to which he also dedicated a recent important volume: "Prodromi di geometria algebrica", in which he describes with great precision and clarity the methods of research of the Italian school of algebraic geometry (methods which are sometimes criticized, often unjustly).

Segre's on the other hand was an eclectic mind. His mathematical work indeed has touched upon many other fields, some more or less bordering on the field of algebraic geometry and others also far removed from that field. I recall, first, his works of differential-projective geometry of varieties and of their transformations, with important ties to algebraic geometry and to the theory of differential equations (Segre dedicated a volume to these subjects too). Other fields of research successfully studied by him concern arithmetical properties on algebraic varieties, the theory of algebraic correspondences, the relations between continuous variation in algebraic geometry and homotopy in topology, the theory of the analytic functions of several complex variables considered from a geometric and topological point of view, and finally fields far removed such as that of hydrodynamics on a sphere, to which Segre dedicated a youthful work in which he achieves, among other things, a mathematical explanation for the formation of anticyclones.

In the last 25 years, Segre was particulary interested in what he called the "geometries of Galois", or "finite geometries". After a brillant introductory volume entitled "Lezioni di geometria moderna" (later translated into English with important additions), Segre investigates these unusual types of geometry in various works, among which four great papers stand out. He moves with great ease in an unexplored field which, for its finitenes is not simpler to deal with than the classical fields, for all or almost all of it has to be constructed *ex novo* (from the first steps of projective geometry), and often the geometric difficulties are transformed into arithmetical ones, although the geometric spirit will always be the weapon of the author's choice.

During these years, if on the one hand various mathematicians in Italy and abroad collaborated with lively interest in the development of these theories (which have already been applied to the field of statistics and to that of information theory), on the other hand other mathematicians have declared their scepticism in the matter. Not by chance did Segre, in dedicating an important paper (entitled "Forme e geometrie hermitiane con particolare riguardo al caso finito") to the memory of the great algebraic geometers G. Castelnuovo and C. Segre, note that the now classic works of the latter on complex and hyperalgebraic geometry did not immediately achieve the recognition they deserved, precisely because of their innovative content.

Here it is not possible to examine the question deeply. Moreover, it can be answered clearly only in the future. In any case, however, it must be said that whoever approaches these works of Segre cannot but recognize once again the author's extraordinary imagination, penetrative force and coordinative ability.

And this same imagination is shown also later in going from research in finite geometry to other research, which is more general in a certain sense, such as the study of incidence structures, involutive correspondences in finite sets, the famous problem referred to as the four-colour problem, which he so brilliantly transforms into a question of Galois geometries, etc.

All these topics, and others which I shall not mention here, more or less closely related to finite geometry, have in common the fact that they deal with questions concerning a finite number of elements. And it must be clarified, as I have already indicated, that this is not a simplification but a cause at times of greater difficulty as soon as the questions that are put are not limited to the first immediate observations. This may seem strange but is not really so for anyone who bears in mind that the idea of "continuum", which arose in the 3rd century B. C. and exploded in all its power two thousand years later, has provided mathematical concepts and instruments suited to the study of an extraordinarily vast number of physical phenomena, which are by their very nature "not continuous". The "continuum" can then be thought of as a sort of "convenient approximation" of the discontinuous, or, better of the "discrete". It is understandable, therefore, that the direct study of the "discrete", or in particular of the "finite", without such an "approximation", presents greater difficulties.

According to the terminology in use today, the subjects that are related essentially to finite sets come within the boundaries of the "Combinatorial Theories". Segre himself, in opening the International Colloquium on such theories (which took place in Rome, in 1973) recognized that "combinatorial theories have at the present moment aspects which are relatively fragmentary and have many gaps". But he added that "the probing and the completion" of these theories "can have important results", and that "it may also furnish mathematics with new means useful for classical lines and for eliminating the divergence which has come about between the more recent and more widely studied themes of mathematical research and those of other sciences, including physics" (2).

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Readers of the mathematical research works of Beniamino Segre admire, as I said, the qualities of imagination, penetration and organization of the author, and I would add, his qualities of tenacity, precision and completeness. And so

(2) Translated from « Atti dei Convegni Lincei », n. 17, (1976).

one might think that these qualities were limited to mathematical research, which was for him a constant need of the spirit, more than a passioned interest.

These qualities in essence were distinctive characteristics of every activity of Segre. For example, in his several historical writings and those on the philosophy of mathematics and more generally of science, these qualities shine forth. Together with them one appreciates also his elegant style and richness of ideas, which make his writings rich in content and suggestive at the same time.

Segre gave his first lecture at the University of Bologna at the age of only 28 years, as I have already said, from the chair of "Geometria superiore" which had been previously held by L. Cremona, founder of the Italian algebraic geometry. He spoke on "Geometry in Italy from Cremona to our day". The subject was exacting and dangerous (as one can easily imagine), but Segre developed it so objectively, brilliantly and elegantly that the lecture was published in the "Annali di matematica" (as seldom happens).

This scientific-cultural masterpiece of Segre's does not remain isolated. From now on he has many occasions to present splendid portraits of mathematicians who are no longer living and of their works (G. Castelnuovo, G. Fubini, G. Peano, F. Severi, C. Segre, F. Enriques, T. Levi-Civita, and many others) as well as of universal scientists (Archimedes, Copernicus, Galileo, Kepler and others).

Nor is there in Segre's work any lack of more exacting writings on the history of mathematics; one of these for example, which is part of the "Cahiers d'Histoire Mondiale", masterfully summarizes a period of 4000 years, from the earliest prodromes of algebra and geometry up to the creation of algebraic geometry. There are also lectures or writings of an essentially philosophical-mathematical nature, such as the one on the questions that modern and future electronic computers raise regarding the human brain. We find also very courteous arguments, such as the one with the famous group of French mathematicians the "Bourbakists". Segre admires them for their fine work of revision, expansion and critical organization of mathematics in extremely abstract form, however with some reservations and noting that Bourbakism had a precursor in G. Peano.

In the many years during which Segre was president of the Academy of the Lincei and the Academy of XL, many of us have admired the extraordinary energy and indomitable courage with which he promoted old and new projects, to all of which he made his own personal contribution. And who does not recall the simple physical energy of Segre, who at every meeting, no matter how long and tiring, was always the last person (though certainly not the youngest) to admit his tiredness?

We now re-read with great interest his terse and significant introductions at the numerous national and international conferences organized by him and his colleagues.

A great achievement of Segre was the important and successful creation of the "Centro Linceo interdisciplinare di scienze matematiche e loro applicazioni",

on the basis of a suggestion made by his colleague G. B. Bonino in 1969 on the occasion of the Lincei Symposium on "Le simmetrie". Segre immediately took up the idea with enthusiasm and elaborated it with unflagging tenacity, so that about two years later, in 1971, the "Centro Linceo" was launched and in August 1977 the law was passed which gave it financial independence. Segre barely witnessed this crowning of his work; two months later he died.

Naturally Segre did not lack for the highest recognitions, both in scientific fields and in those of general culture. I have already mentioned some of these awards. I wish to point out also that he was a member of the "Accademia dei Lincei" from 1947 and of the "Accademia dei XL" from 1959; also he was a member of the "Pontificia Academia Scientiarum" and of many other Italian Academies. He was a foreign member of the "Societé Royal des Sciences de Liège", of the "Académie Royal de Belgique", of the "Toulouse Académie des Sciences, Inscriptions et Belles-Lettres", honorary member of the "London Mathematical Society" and of the "Academia Nacional de Ciencias Exactas, Fisicas y Naturales" of Buenos Aires; and finally in 1974 he was elected "Membre correspondant" of the "Académie des Sciences de l'Institut de France".

Beniamino Segre also received numerous prizes and honours, among which: Gold Medal of the "Società Italiana delle Scienze" (1925-30), Gold Medal for "Benemeriti della Scuola, della Cultura e dell'Arte" (1962), Gold Pen of the "Presidenza del Governo Italiano" (1965), "Cavaliere di Gran Croce della Repubblica Italiana" (1969), "Commandeur de la Légion d'Honneur de la République Française" (1974), etc.

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If it is difficult to express a scientific judgment of a scientist who has died recently, it is more difficult to express a human judgment.

Some people felt that Beniamino Segre's was a rather cold character with regard to human affairs. Is this judgment correct?

I do not think so. The "cold" nature, which it might be more accurate to call "rough" or "gruff" (as I have said), I believe was nothing but the external manifestation of an inner timidity, which is in itself a sign of modesty, a trait which is the more appreciable the greater are the qualities of the person.

I should like, in conclusion, to support this valuation of mine, not with a long speech, but by describing — with your permission — a brief episode which goes back about thirty years and which sheds light not only on the human sensitivity of Segre but also on other aspects of his intimate feelings.

Beniamino Segre had been with his family in Rome for a short time, summoned from Bologna after his return to Italy. He had found rather unconfortable living accomodation on the sixth floor of a huge building in the suburbs in a very

small apartment (one of his sons had to sleep in the dining room). But Segre's choice had been dictated, as on other occasions, by his innate modesty and his scientific interest, which was to predominate over every material consideration. And now in fact, he is again close to F. Severi, and the "Istituto di Alta Matematica" in full activity constitutes a powerful catalyzer for scientific research.

A younger colleague and friend, briefly visiting Rome during the Christmas holidays, went with his wife to visit the Segre family. When the young couple, after taking their leave, were passing in the street below, they heard their friend Beniamino call to stop them; he wanted to give them a package of toys for their children which he had forgotten during their visit.

I cannot forget this man, small in physical stature, great in mind and spirit, who that evening hastened down prompted by such kind and tender feeling.